How to Hedge if the Payment Date is Uncertain?

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Abstract

This paper investigates how firms should hedge price risk when payment dates are uncertain, a frequent problem in practice. It derives variance-minimizing strategies and shows that the instrument choice is essential for this hedging problem. A first setting concentrates on hedging with futures. The analysis is then extended to a setting with non-linear exotic derivatives written on futures, allowing for tailor-made payoff functions. In both settings, firms should take positions in derivatives contracts with different maturities simultaneously if the last potential payment date exceeds the remaining maturity of the shortest-maturity derivative contract. An empirical analysis for commodity prices and exchange rates shows that in both settings the optimal strategy clearly outperforms heuristic strategies that use only a single hedging instrument at a time.

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Abstract

This paper investigates how firms should hedge price risk when payment dates are uncertain, a frequent problem in practice. It derives variance-minimizing strategies and shows that the instrument choice is essential for this hedging problem. A first setting concentrates on hedging with futures. The analysis is then extended to a setting with non-linear exotic derivatives written on futures, allowing for tailor-made payoff functions. In both settings, firms should take positions in derivatives contracts with different maturities simultaneously if the last potential payment date exceeds the remaining maturity of the shortest-maturity derivative contract. An empirical analysis for commodity prices and exchange rates shows that in both settings the optimal strategy clearly outperforms heuristic strategies that use only a single hedging instrument at a time.
1. Introduction

Derivatives markets offer hedging opportunities for a variety of different risks, such as commodity price risk, interest rate risk, or foreign exchange risk. The design of a firm’s hedging strategy, however, is often complicated by another source of risk: the uncertainty about the timing of cash flows to be hedged. This kind of uncertainty arises, for example, if a firm produces for stock but does not know exactly when its products will be sold, technological problems or weather conditions lead to uncertain production times, a claim of recourse in foreign currency is decided in a lawsuit that may take more or less time, or ongoing negotiations about an order make it uncertain when raw materials will be required by a producing firm.\(^1\)

In this paper, we study the corresponding problem of hedging price risk when the payment date is uncertain and show that this hedging problem raises interesting issues about the instrument choice.\(^2\) In particular, we investigate the following two research questions: When the payment date is uncertain, should hedging strategies use a mix of different derivative contracts with different maturities or is it sufficient to use one maturity at a time? Is hedging with linear contracts such as forwards or futures sufficient or should firms use options, possibly even exotic ones? These questions are studied both theoretically and empirically.

As a theoretical contribution the paper derives variance-minimizing hedging strategies in two settings. The first setting considers hedging with futures contracts only and the second one allows for exotic options written on futures, leading to tailor-made payoff functions. Our main result for the first setting is that under perfectly

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1 A concrete example of uncertainty about the timing of cash flows to be hedged are the firm-flexible contracts sold by Metallgesellschaft (MG) (Mello and Parsons, 1995; Pirrong, 1997). Under firm-flexible contracts, buyers were allowed to defer or accelerate purchases, but had to buy all quantities deferred by the end of the contract. This feature was one of the issues MG’s hedging strategy had to face, which finally ended in a disaster for the firm.

2 The timing option given to the holder of a short position in futures contracts (Boyle, 1989) provides a means to deal with such uncertain payment dates. However, a timing option can usually be exercised only during the maturity month. If uncertainty about the timing of cash flows refers to longer periods, the problem remains that it is unclear which maturities to choose.
liquid futures markets firms should simultaneously hold futures contracts of different maturities if the last potential payment date exceeds the remaining maturity of the shortest-maturity derivative contract. Over time the hedge is adjusted to account for new information about the payment date. Simultaneous coverage of different maturities is also required in the second setting. Non-linear derivatives written on the prices of multiple futures are optimal if uncertainty about the payment date and price risk are correlated, since such correlation leads to profits being non-linearly related to price. If the two sources of risk are independent, linear contracts are optimal.

Our paper also makes an empirical contribution by comparing the performance of different hedging strategies in the presence of uncertain payment dates. The empirical study is based on the prices of oil, copper, and gold as well as the US Dollar (USD) to Euro exchange rate. It compares the optimal strategy in each setting with various alternatives, including heuristic strategies that hold only positions in a single futures contract at a time. Our empirical results show that the optimal hedging strategy clearly outperforms such heuristic alternatives often used in practice. For short hedge horizons and a weak dependence between price and payment date, linear hedging instruments are sufficient. If price and payment date are strongly dependent, however, non-linear derivatives lead to a significant improvement in terms of risk reduction.

Our paper belongs to the vast literature on corporate hedging decisions. This literature includes very general dynamic models (e.g., Bolton et al. (2011) and Rampini et al. (2014)) of the investment, financing, and risk management policies of financially constraint firms. We focus on one specific aspect: a firm’s hedging policy with derivatives. Moreover, we abstract from liquidity constraints that might restrict the firm’s usage of derivatives (see also Lien (2003) and Adam-Müller and Panaretou (2009)). Our model is more general, however, than the models by Bolton et al. (2011) and Rampini et al. (2014) with respect to two aspects that are essential for
our research questions. First, it treats the payment date of the firm’s cash flows from operations as a stochastic state variable. Second, it allows for the use of derivative contracts with different maturity dates, that is, it is less restrictive with respect to the instrument choice.

The issue of uncertain payment dates is closely related to two other problems that have been studied in the literature on corporate hedging, namely, basis risk and quantity risk. Basis risk may occur for different reasons. For example, the grade of a commodity may differ from the grade a futures contract relates to or a futures contract with the desired maturity date may just not be available. The latter problem was termed an “imperfect time hedge” in the literature (Batlin, 1983; Karp, 1988). The imperfect time hedge is similar to our hedging problem in the sense that an unknown (stochastic) payment date of the cash flows to be hedged introduces a potential mismatch between the payment date and the maturity dates of hedging instruments, that is, leads to basis risk. However, the reasons why basis risk occurs are different. In the imperfect time hedging problem the payment date is known but no derivative contract is available that matures at that date. In our problem the payment date is unknown (stochastic) but multiple derivatives contracts exist whose maturity dates cover all potential payment dates. These differences in setting have important consequences: The hedging problem we study mandates an analysis of multiple periods and it turns out that the optimal strategy uses multiple hedging instruments with different maturity dates simultaneously. This is not the case for the imperfect time hedging problem, as studied by Batlin (1983) and Karp (1988).

Another way of looking at our hedging problem is that, at any potential payment date, a promised cash flow may or may not occur, which means that the quantity to be hedged is either that promised or zero. Thus, we have to deal with a certain kind of quantity risk. Such quantity risk could result either from demand uncertainty

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3 The effects of basis risk on hedging strategies with futures have been analyzed by, for example, Rolfo (1980), Anderson and Danthine (1980, 1981), Benninga et al. (1984), Briys et al. (1993), and Adam-Müller and Nolte (2011).
(Leland, 1972) or production risk.\footnote{Hedging problems with futures contracts under both price risk and quantity risk have been analyzed by, for example, Benninga et al. (1985), Eaker and Grant (1985), Kerkvliet and Moffett (1991), Lapan and Moschini (1994), Chowdhry (1995), Adam-Müller (1997), and Brown and Toft (2002).} What is specific about the hedging problem when payment dates are uncertain, however, is again the specific dynamic setting. In this setting a variance-minimizing hedging strategy requires to hold derivatives with different maturities simultaneously, an issue of the instrument choice that has not been investigated in the literature on basis risk or quantity risk so far. For this reason, our analysis extends this strand of literature.

Finally, our paper belongs to the studies that investigate which payoff profiles of derivatives are optimal for hedging. The seminal work by Brown and Toft (2002) addresses this issue in a one-period setting with price risk and quantity risk and Korn (2010) extends the analysis by relaxing the distributional assumptions. Mahul (2002) and Chang and Wong (2003) study the effect of basis risk on the usage of futures and options and Moschini and Lapan (1995) investigate this problem under joint price, basis, and quantity risk. The impact of counterparty credit risk and liquidity risk on the hedging roles of futures and options is studied by Mahul and Cummins (2008) and Adam-Müller and Panaretou (2009), respectively. We are the first, however, to address the issue of optimal payoff functions of derivatives when payment dates are uncertain. Reflecting the specific structure of this problem, we show that options should be written on multiple underlyings, referring to futures with different times to maturity. This aspect of the instrument choice has not been documented in the literature so far.
2. Optimal Hedging Policies

2.1. Setting

Consider a firm with a single product.\(^5\) The date is currently date 0; the output quantity, \(Q\), is already determined,\(^6\) and the product will be sold at a later date. However, the exact timing of the sale is uncertain. It could take place on either date 1 or date 2, with date 2 being the firm’s planning horizon.\(^7\) The exogenous product prices\(^8\) \(P_1\) and \(P_2\) at dates 1 and 2, respectively, are uncertain and the costs \(C\) are deterministic.\(^9\) The firm has the opportunity to finance or invest at a fixed rate \(r\) and invests all revenues that accrue prior to the planning horizon. Under these assumptions, the firm’s operating profits over its planning horizon equal

\[
P_1 Q I (1 + r) + P_2 Q (1 - I) - C,
\]  

(1)

where \(I\) is a random variable describing the uncertain timing of the sale. It takes a value of one if the sale occurs at date 1 and a value of zero otherwise.\(^10\) The (subjective) probability for a sale at date 1 is denoted by \(q\). As we show later, the

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5 The analysis can also be extended to a firm with multiple products. The resulting hedging strategies are very similar in structure. Detailed results are available on request.

6 The model can easily be extended to incorporate quantity risk, that is, the output quantity could be random.

7 A setting with three dates is mainly used for ease of exposition. However, the framework can easily be extended to include \(N\) further potential selling dates between the current date and the firm’s planning horizon. Under this extended framework, an optimal hedging strategy that is analogous to the one presented in Proposition 1 can be derived.

8 The price variable could also be an exchange rate. In this case, \(Q\) represents an exporting firm’s revenues in a foreign currency.

9 In our setting, the magnitude of such deterministic costs is actually irrelevant for the optimal hedging strategy. However, the model can easily be extended to include stochastic costs. If these costs are correlated with prices, they will affect the firm’s hedging strategy.

10 The variable \(I\) could also be a choice variable of the firm. The decision to sell the product at date 1 or to defer the sale to date 2 could depend, for example, on the product price at date 1. As we show, such dependencies have an impact on the instrument choice. From the perspective of date 0, however, \(I\) is a random variable, reflecting all information available at date 0.
firm has to specify this probability to implement the optimal hedging strategy.\textsuperscript{11} It is likely that $q$ depends on the output quantity $Q$, for example, the higher $Q$ the longer it might take to sell it. Therefore, $q$ is meant to be a conditional probability, conditional on the output level.

The firm has access to derivative contracts written on prices\textsuperscript{12} that it can use to hedge the risk in its operating profits. Contracts with different maturity dates (date 1 and date 2) are available\textsuperscript{13} and can be entered into at date 0. At date 1 derivatives positions can be adjusted in response to new information available.\textsuperscript{14} Thus, we have a situation in mind where payments from operations may potentially occur after the expiration date of the shortest-maturity derivative contract.\textsuperscript{15} The terms $g_1(\cdot)$ and $g_2(\cdot)$ denote the corresponding price-contingent payments arising from derivatives at dates 1 and 2, respectively. All derivatives are deferred payment contracts, that is, any premium is paid at maturity. Cash flows from derivatives that accrue at date 1 are either invested or financed at the rate $r$, leading to the following profits from the derivative contracts:

$$g_1(\cdot)(1 + r) + g_2(\cdot). \tag{2}$$

Combining profits from operations and profits from derivatives then yields the

\textsuperscript{11} Fortunately, the optimal strategy shows some robustness with respect to imprecise estimates of the probability $q$. Presume that the firm uses the estimate $\hat{q}$ instead of the true parameter $q$. Then the optimal strategy (according to Corollary 1 on page 11) implemented with $\hat{q}$ still leads to a lower variance than a rollover strategy (see p.17) if $\hat{q} > 2q - 1$ and to a lower variance than a long-term strategy (see p.17) if $\hat{q} < 2q$. The proof is available on request.

\textsuperscript{12} Which prices are relevant will become apparent when we derive the optimal hedging strategies.

\textsuperscript{13} Note that under this assumption maturity dates exactly match potential payment dates. In reality, contracts with the required maturities may not be available. This leads to an additional imperfect time hedging problem as studied by Batlin (1983) and Karp (1988), introducing additional basis risk.

\textsuperscript{14} As a special case, the firm could take zero derivative positions at date 0 and adjust this (zero) positions as such that it takes non-zero positions for the first time at date 1.

\textsuperscript{15} If all potential payment dates would fall within the time to maturity of the shortest-maturity derivative contract, it would be sufficient to hold hedging positions in this contract and to use the timing option to deal with any uncertainty about the payment date.
following total profits:

\[ \Pi = P_1 Q I (1 + r) + P_2 Q (1 - I) - C \]

\[ + g_1(\cdot)(1 + r) + g_2(\cdot). \]  \hfill (3)

The firm’s hedging problem is to find appropriate payoff functions \( g_1(\cdot) \) and \( g_2(\cdot) \). Optimal hedging policies are derived under two additional assumptions.

**ASSUMPTION 1:** *The expected profit from derivatives contracts is zero for all contracts and all periods.*

Under this assumption, derivatives do not offer the firm opportunities to earn risk premiums or to time the market.\(^\text{16}\) Whether this is a reasonable assumption depends on the particular market, the particular derivative, and the firm’s information set. At the very least, Adam and Fernando (2006) and Brown et al. (2006) provide empirical evidence that selective hedging (market timing) by non-financial firms is unsuccessful, on average, and Bartram (2017) generally finds little evidence of selective hedging with derivatives for an international sample. Therefore, the assumption of zero expected profits from selective hedging is the natural starting point to derive hedging strategies. However, Adam and Fernando (2006) also find evidence for stable risk premiums earned by gold mining firms on their derivatives positions. Such risk premiums could be integrated in our setting by allowing for non-zero expected returns of derivatives positions. As shown by Anderson and Danthine (1981), non-zero expected returns lead to an additional component in the firm’s derivatives position in the mean-variance framework. The derivatives position then consists of a “pure hedge component” (position when expected profits of derivatives are zero) plus a “speculative component” (position that an investor (speculator) with no price exposure to be hedged would take). However, because this result is known and the

\(^{16}\) Note that Brown and Toft (2002) make the same assumption in their analysis of optimal hedging policies.
inclusion of this second component does not add to the contribution of our paper, we concentrate on the pure hedge in our analysis.

Despite risk premiums, other features of real derivatives markets are transaction costs and the illiquidity of certain contracts. In particular, there is a lack of liquidity in many deferred contract months in Western futures markets. Assumption 1 does not capture this observation. If long-term contracts have higher (direct or indirect) costs of illiquidity than short-term contracts, they should have lower expected returns from the perspective of the hedging firm. Assuming zero expected returns for all derivatives contracts can therefore be interpreted as a joint assumption of no risk premiums and perfect liquidity of all contracts. Differences in liquidity between short-term and long-term contracts could be integrated in our setting by allowing for non-zero expected returns of derivatives positions (due to illiquidity costs) in a mean-variance framework. In such a framework, the qualitative effects of illiquidity are clear. The less liquid derivatives are, the less firms should hedge. Moreover, the relative weights of less liquid and more liquid contracts should be adjusted.\footnote{Interestingly, for many Japanese commodity futures contracts, the longer-term contracts are the more liquid ones (Ciner, 2002).}

In the extreme, if long-term contracts are very illiquid, they may not be used at all and a dynamic strategy with short-term contracts only (stacked hedge) could be optimal in this extended setting. A full quantitative treatment of this setting is left for further research.

ASSUMPTION 2: \textit{The firm minimizes the variance of total profits.}

Variance minimization is a common starting point when risk reduction needs to be made concrete.\footnote{Of course, variance minimization can be criticized on theoretical grounds. One aspect is that variance minimization is compatible with expected utility maximization only under certain assumptions about utility functions and price distributions. Moreover, variance minimization could lead to time inconsistencies in multi-period problems (for a discussion of the latter issue, see, e.g., Basak and Chabakauri (2010)). Under our assumptions, however, the resulting hedging strategy is time consistent.} Its major advantage lies in the tractability of the corresponding
optimization problem, which often leads to closed-form solutions that are easy to interpret and implement.

2.2. Optimal Linear Hedge

We start our analysis of hedging strategies by concentrating on linear hedging instruments, such as forwards or futures.\textsuperscript{19} At date 0, the firm has access to a short-term futures (expiring at date 1) and a long-term futures (expiring at date 2). At date 1, the firm can adjust its hedging position in response to new information, that is, it can change its position in the long-term futures. The short-term contract makes the following payment at date 1:

\[ g_1(\cdot) = (P_1 - F_{0,1}) h_{0,1}, \]

where \( F_{0,1} \) is the futures price of the short-term contract at date 0 and \( h_{0,1} \) denotes the number of contracts bought. The long-term contract makes the following payment at date 2:

\[ g_2(\cdot) = (F_{1,2} - F_{0,2}) h_{0,2} + (P_2 - F_{1,2}) h_{1,2}, \]

where \( F_{0,2} \) and \( F_{1,2} \) are the futures prices of the long-term contract at dates 0 and 1, respectively. \( h_{0,2} \) denotes the number of (long) futures position held over the first period (0 to 1) in the long-term contract and \( h_{1,2} \) is the (net) position held over the second period (1 to 2).\textsuperscript{20} Given the payments from futures according to

\textsuperscript{19} We do not distinguish between forwards and futures here and use the terms interchangeably. In particular, we do not consider a marking to market.

\textsuperscript{20} Accordingly, the change in positions in the long-term contract made at date 1 equals \( h_{1,2} - h_{0,2} \), that is, the firms buys \( h_{1,2} - h_{0,2} \) contracts at date 1 if this expression is positive or sells \( h_{1,2} - h_{0,2} \) contracts if it is negative.
Equations (4) and (5), the firm’s total profits over the planning horizon become

\[ \Pi = P_1 Q I (1 + r) + P_2 Q (1 - I) - C 
+ (P_1 - F_{0,1})(1 + r) h_{0,1} + (F_{1,2} - F_{0,2}) h_{0,2} + (P_2 - F_{1,2}) h_{1,2}. \]  

Given the total profits from Equation (6), the firm has to solve the optimization problem

\[ \min_{h_{0,1}, h_{0,2}, h_{1,2}} \text{Var}_0[ \Pi ]. \] (7)

Variance minimization according to Equation (7) poses a dynamic optimization problem that has to be solved recursively, using the information available on the two decision dates. This problem is solved in the Appendix. The following proposition provides the solution.

PROPOSITION 1: When the payment date is uncertain, the optimal hedging policy with linear contracts is given by

\[ h_{1,2}^* = -Q (1 - I), \] (8)

\[
\begin{pmatrix} h_{0,1}^* \\ h_{0,2}^* \end{pmatrix} = -Q \begin{pmatrix} q \\ 1 - q \end{pmatrix}
+ \begin{pmatrix} \text{Var}_P & \text{Cov}_{PF} \\ \text{Cov}_{PF} & \text{Var}_F \end{pmatrix}^{-1} \begin{pmatrix} \text{CoSk}_{PI} \cdot (F_{0,1}(1 + r) - F_{0,2}) \\ \text{CoSk}_{FI} \cdot (F_{0,1}(1 + r) - F_{0,2}) \end{pmatrix}
+ \begin{pmatrix} \text{Var}_P & \text{Cov}_{PF} \\ \text{Cov}_{PF} & \text{Var}_F \end{pmatrix}^{-1} \begin{pmatrix} \text{CoSk}_{PPI} - \text{CoSk}_{PI} \\ \text{CoSk}_{PF} - \text{CoSk}_{FII} \end{pmatrix},
\]

where \( \text{Var}_P, \text{Var}_F, \) and \( \text{Cov}_{PF} \) are the variances and covariance of \( P_1 (1 + r) \) and \( F_{1,2}; \) and \( \text{Cov}_{PI} \) and \( \text{Cov}_{FI} \) denote the covariances between the payment date in-
indicator $I$ and the two price variables $P_1(1+r)$ and $F_{1,2}$, respectively. Finally, the terms $CoSk_{PPI}$, $CoSk_{PFI}$, and $CoSk_{FFI}$ are measures of coskewness defined as $CoSk_{PPI} := E[(P_1(1+r) - F_{0,1}(1+r))^2(I-q)]$, $CoSk_{PFI} := E[(P_1(1+r) - F_{0,1}(1+r))(F_{1,2} - F_{0,2})(I-q)]$, and $CoSk_{FFI} := E[(F_{1,2} - F_{0,2})^2(I-q)]$.

Proposition 1 provides several insights into the structure of optimal futures hedges under both price risk and uncertainty about the payment date. (i) The quantity $Q$ is just a linear scaling factor of the hedge positions. (ii) The hedge position $h_{1,2}^*$ at date 1 guarantees that no risk remains in the second period. If the sale has already been made at date 1, there is no price risk and $h_{1,2}^* = 0$. If the sale will occur at date 2, the remaining price risk will be fully hedged and $h_{1,2}^* = -Q$. (iii) The hedging strategy followed at date 0 is more complicated. It uses both futures contracts simultaneously, that is, positions in two derivatives contracts with different times to maturity are taken to hedge the risk inherent in one payment. This characteristic of the optimal strategy clearly distinguishes it from heuristic alternatives that are used in practice to deal with uncertain payment dates. The reason why the firm already engages in hedging activities at date 0—despite the fact that the exact payment date is still unknown—is that the firm is already exposed to price changes over the first period (from $t = 0$ to $t = 1$). If this exposure is left unhedged, hedging effectiveness will be largely reduced, as we demonstrate in Section 3. (iv) The optimal strategy requires to adjust positions held in long-term futures ($h_{0,2}^* \neq h_{1,2}^*$) at date 1 and is therefore a truly dynamic strategy. The reason for this adjustment is that new information about the payment date is revealed at $t = 1$, that is, the firm can then make a better-informed hedging decision. (v) The optimal hedge position at date 0 comprises three components. The first component consists of the unconditional probabilities of a sale at date 1 ($q$) and date 2 ($1-q$) multiplied by $-Q$. This result is intuitively appealing, because a high probability of a sale at date 1 makes the short-term future the natural instrument to use. To
the contrary, if the probability of a sale at date 2 is high, the long-term future is
the natural hedging instrument. (vi) In addition, the firm could try to hedge some
of the uncertainty about the payment date with futures written on prices. Such an
attempt shows up in the second component. It can be interpreted as the vector of
regression coefficients from a multiple linear regression of \( I(F_{0,1}(1 + r) - F_{0,2}) \) on \( P_1 \)
and \( F_{1,2} \). Note that \( F_{0,1}(1 + r) - F_{0,2} \) is the date 0 expectation of \( P_1(1 + r) - F_{1,2} \),
which is the firm’s remaining exposure to the randomness of the payment date,
given the optimal hedging strategy in the second period. (vii) The third component
involving the coskewness terms arises, because the link between the price and the
payment date indicator \( I \) is multiplicative. (viii) In summary, setting up the optimal
linear hedging strategy at date 0 can be split into the following three steps: (1) Use
the expected exposure to price risk in the first period and hedge it fully, (2) use
the expected exposure to payment date uncertainty in the first period and hedge
it as much as possible with linear instruments written on prices, and (3) account
appropriately for the fact that there is a multiplicative link between price risk and
the uncertainty about the payment date.

As a direct implication of Proposition 1, Corollary 1 shows that a substantial
reduction in the complexity of the optimal hedging strategy results if uncertainty
about the payment date is independent of price risk. Such a case is realistic un-
der certain circumstances, such as when an uncertain payment date results from a
project’s technological problems.

COROLLARY 1: When the payment date is uncertain, if price and payment date
are independent, the optimal hedging policy with linear contracts is given by

\[
 h_{1,2}^* = -Q (1 - I) , \quad h_{0,1}^* = -Q q , \quad h_{0,2}^* = -Q (1 - q) .
\]  

(10)

Compared to the general case, the hedge positions at date 0 do not depend on
any characteristic of the joint distribution of $P_1$ and $F_{1,2}$. Moreover, positions in both futures are generally sell positions and the firm sells exactly $Q$ contracts in total. In this sense, the firm’s output is fully hedged.

### 2.3. Optimal Exotic Hedge

Now we lift the restriction to linear hedging instruments and allow the firm to enter into derivatives with any linear or non-linear payoff structure (exotic hedges). At date 1, derivative contracts (exotic options) written on long-term futures are available, that is, options written on the price $P_2$, the only risk remaining at that time. At date 0, the firm can enter into (exotic options) contracts written on the prices of both short-term and long-term futures at the end of the period, namely, $P_1$ and $F_{1,2}$. Under these assumptions, the firm’s optimization problem becomes

$$\min_{g_1(P_1, F_{1,2}), g_2(P_2)} \text{Var}_0[\Pi], \quad \text{s.t.} \quad E_0[g_1(P_1, F_{1,2})] = E_1[g_2(P_2)] = 0. \quad (11)$$

Note that problem (11) is a dynamic functional optimization problem, which requires finding whole payoff functions instead of single parameters. The following proposition states the solution to the problem. A proof is provided in the Appendix.

**Proposition 2:** When the payment date is uncertain, the optimal exotic hedge is given by

$$g_2^*(P_2) = -Q (1 - I)(P_2 - F_{1,2}), \quad (12)$$

$$g_1^*(P_1, F_{1,2}) = -Q \left[ q_{|P_1,F_{1,2}} P_1 + (1 - q_{|P_1,F_{1,2}}) F_{1,2} (1 + r)^{-1} \right] - k, \quad (13)$$

where $q_{|P_1,F_{1,2}}$ denotes the conditional probability (given $P_1$ and $F_{1,2}$) of a sale at date 1 and $k$ is a constant parameter ensuring that the expected profit of the derivatives contract is zero.
Some properties of the optimal exotic hedge are worth mentioning: (i) The optimal derivative position in the second period is still the same linear hedge as in Proposition 1. The reason is that such a hedge already eliminates risk completely in the second period and no improvement is possible. (ii) The crucial element of the hedge in the first period is the conditional probability \( q_{|P_1 F_{1,2}} \) of a payment at date 1. This conditional probability needs to be specified by the firm. (iii) The optimal hedge will generally deviate from the optimal linear hedge of Proposition 1, because the conditional probability \( q_{|P_1 F_{1,2}} \) can be a complex non-linear function of the prices. Therefore, uncertain payment dates provide a reasoning for the use of non-linear derivatives. (iv) If price and payment date are independent, however, the conditional probability equals the probability \( q \), which is an unconditional probability in the sense that it does not use any information on the prices \( P_1 \) and \( F_{1,2} \). Thereby, we obtain a substantial reduction in complexity, as stated in Corollary 2.

**Corollary 2:** When the payment date is uncertain, if price and payment date are independent, the optimal exotic hedge is the linear hedge, as given in Corollary 1.

Corollary 2 stresses the importance of the rather simple linear hedging strategy that combines positions in a short-term future and a long-term future according to the probability of an early sale. If price and payment date are independent, such a strategy is not only the optimal linear hedging policy but also the best one generally accessible with derivatives. Therefore, if firms do not consider dependencies between prices and the timing of cash flows in their hedging decisions, this result supports the major role of linear instruments in risk management.

The following example illustrates optimal exotic hedges when price and payment date are dependent. We parameterize the conditional probability of a payment at date 1 as a logistic function of the product’s relative price change in the first period,
\[ q_P = \frac{\exp(-\beta(P_1 - P_0))}{1 + \exp(-\beta(P_1 - P_0))}, \]  

where \( \beta \) is a parameter governing dependence. The rationale behind Equation (14) is as follows: The potential buyer of the product conditions the decision to buy early (date 1) upon assessment of the price being rather high or rather low at date 1. If the price has increased in the previous period, it will be judged as relatively high and the purchase will have a higher probability of being deferred. If the price has decreased, it will be seen as currently favorable and an immediate purchase will have a higher probability.\(^{21}\)

We consider three scenarios with different parameters values for \( \beta \), using an initial price \( P_0 = 1 \). For \( \beta = 0 \), price and payment date are independent and the probability of a sale at date 1 equals 0.5. A \( \beta \) value of four represents moderate dependence. In this case, if the price has risen by 10\% in the previous period, the probability of an immediate sale will only be 0.4. With \( \beta = 8 \), we have strong dependence, meaning that a price increase of 10\% results in a conditional probability of about 0.2. Figure 1 shows the optimal payoff functions for the three scenarios.

[ Insert Figure 1 about here ]

The upper part of Figure 1 refers to the case of independent price and payment date. It confirms that the optimal hedge has payoffs that are linear in both price variables. The middle and lower parts of the figure illustrate the effects of dependence. Since the conditional probability in Equation (14) does not depend on \( F_{1,2} \), the optimal payoff function is linear in \( F_{1,2} \) for every fixed value of \( P_1 \). However, we observe strong non-linearities in \( P_1 \). The optimal payoff function is increasing with \( P_1 \) in some regions and decreasing in others. Moreover, it has both concave and

\(^{21}\) This example argues from the perspective of the buyer of the product, causing demand uncertainty for the firm. If \( I \) is a decision variable of the firm, however, the reasoning would be reversed and the conditional probability for a sale at date 1 should be higher if the price has increased and is seen as currently favorable by the firm.
convex areas. The specific form of the payoff function can be better understood if we consider an extremely high beta. In this case, we can be confident that a value of $P_1$ below one almost surely leads to an immediate sale, making a full hedge with short-term futures the best way to proceed. Conversely, for a value of $P_1$ above one, the sale will almost surely take place at date 2 and a full hedge with long-term futures is optimal. This is the reason for the “kink” at $P_1 = 1$ in the lower part of Figure 1.

3. Hedging Performance

It has been shown that the strategy set forth in Corollary 1 will have the lowest variance if price risk and uncertainty about the payment date are independent, and that the strategy formulated in Proposition 2 will minimize variance in the case of price-payment date dependence. It has not yet been determined, however, by how much these policies will outperform alternatives in a real world context. We therefore investigate hedging performance: In a first step, we look at the case of independent risks. Here we compare the hedging effectiveness of the optimal strategy over different markets and examine by how much it improves on alternative heuristic hedging strategies that are used in practice. In particular, these heuristic strategies do not hold futures with different maturity dates simultaneously, an important characteristic of the optimal strategy. In a second step, we introduce dependence between payment date and price and analyze customized non-linear contracts. Again, we compare hedging performance to some heuristics, including the optimal linear strategy of Proposition 1. By doing so, we provide empirical evidence on the two research questions about the instrument choice in the presence of uncertain payment dates that we address in of this paper.
3.1. Data and Design

To investigate the empirical performance of hedging strategies, we must specify the firm’s planning horizon, price risk, and the uncertainty about payment dates. For our base case, we assume that the firm’s planning horizon is one year, which seems appropriate, since empirical evidence shows that many firms predominantly use derivatives with maturities of one year or less.\footnote{Compare the survey results of Bodnar et al. (1998) and Bodnar and Gebhardt (1999) for non-financial firms in the United States and Germany. Adam et al. (2017) state that, in their sample of gold mining firms, hedging activity decreases sharply with the hedge horizon and mainly concentrates on next year’s production.} This evidence could indicate that longer-term exposure is managed differently, for example, by means of operational hedging. It is further assumed that the planning horizon consists of two periods of equal length and revenues can occur at the end of each; that is, in the base case, they occur either after six months or after 12 months.

No particular distributional assumption is made to capture price risk. Instead, our study uses 25 years of historical price data from several commodity markets and the FX market. In particular, we use West Texas Intermediate (WTI) crude oil futures, copper futures, gold futures, and USD–Euro spot and forward exchange rates. Oil, copper, and gold are selected because the role of convenience yield and convenience yield risk is quite different for the three commodities. As Schwartz (1997) and Casassus and Collin-Dufresne (2005) show, convenience yields are most important for oil and least important for gold. A time-varying convenience yield constitutes an important risk factor for commodity derivatives in addition to the spot price and could strongly affect hedging performance. For FX derivatives, the differential between interest rates in the two currency regions is the crucial risk factor beyond the spot exchange rate. For all commodities, we use monthly futures prices of contracts with appropriate maturities from January 1990 to January 2015. The oil futures we use trade on the New York Mercantile Exchange (NYMEX) and the copper and gold futures on the New York Commodities Exchange (COMEX).
data were supplied directly by the Chicago Mercantile Exchange (CME) group. The selected dates within a month refer to the last trading day of the expiring futures contract. Therefore, the price of the expiring contract is a sensible proxy for the spot price. The spot and forward exchange rates of the USD versus the Euro\textsuperscript{23} were obtained from Thomson Reuters Datastream and also cover the period between January 1990 and January 2015.

Uncertainty about the payment date is integrated in two different ways: In a first setting, it is assumed to be independent from price risk and only linear contracts are considered. Here, the probability $q$ of a sale taking place at the end of the first period does not depend on the price at date 1. In a second setting, dependence has to be modeled, which will be done in the same way as illustrated in Section 2.3. We use the function from Equation (14) to describe the conditional probability (conditional on the price at date 1) of an early sale. By setting the parameter $\beta$ equal to four and eight, we generate two scenarios, with moderate and high dependence levels, respectively.

This study considers six different hedging strategies: 1) *Optimal hedge:* This is the variance-minimizing strategy established in Proposition 2. 2) *Optimal linear hedge:* This is the linear variance-minimizing strategy established in Proposition 1. If price risk and payment date risk are independent, this strategy is identical to strategy 1. 3) *Unconditional linear hedge:* This hedging strategy uses the unconditional probability $q$ to determine the positions held in futures. If price risk and the uncertainty about the payment date are independent, this strategy is identical to strategies 1 and 2. 4) *Rollover hedge:* Under this strategy, a full hedge in the six-month futures is placed at date 0. If the sale occurs at the end of the first period, no new contract is entered into at $t = 1$. If the sale does not occur at date 1, the futures position is rolled over into the next six-month contract. 5) *Long-term hedge:* With such a strategy, the firm adopts a full hedge position in a 12-month futures at

\textsuperscript{23} Prior to the introduction of the Euro, exchange rates based on the Deutschmark were used.
date 0 and, thus, the maturity date of the contract coincides with the last possible
date of sale for the product. The futures position remains unchanged until date 2
if no sale occurs at date 1. If the product is sold at date 1, the hedge position is
closed out by taking an offsetting long futures position. 6) Deferred hedge: Under
this strategy, hedging will be deferred until the exposure is known exactly. In other
words, no futures contract will be entered into at $t = 0$. If the sale occurs at the end
of the first period, no futures will be needed for the second period either. Only if
the sale has not taken place will a full futures hedge be entered into for the second
period. As a reference point, we also consider that the firm does not hedge at all.
The risk of an unhedged position will be used to assess the hedging effectiveness of
the optimal strategy for different markets.

For each market and each month in the data period, we calculate percentage
returns of spot and futures contracts. Starting from prices observed at the end of
our data period (January 2015), we use these returns to create scenarios for the
development of spot and futures prices. Because six-month futures do not trade
every month for gold and some values are missing for oil and copper, the number of
scenarios varies between the markets. We have 291 scenarios for oil, 293 for copper,
147 for gold, and 295 for the exchange rate. From these price scenarios, the variances
for the different strategies are finally calculated. 24

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24 Closed-form expressions for the profit variances based on specific sample moments exist for all
hedging strategies and the no-hedge strategy. Formulas and proofs are available on request.
3.2. Results: Independent Payment Date and Price

3.2.1. Hedging Effectiveness

To measure the risk reduction achieved by the optimal hedging strategy, we use a slightly altered version of what is known as the Johnson measure:

\[ 1 - \frac{\text{standard deviation of hedged position}}{\text{standard deviation of unhedged position}}. \] (15)

This simple measure of hedging effectiveness, which fits well with the goal of variance minimization, calculates the volatility reduction of profits achieved by the optimal strategy relative to an unhedged position.

[Insert Table 1 about here]

The results presented in Table 1 confirm that the optimal strategy can indeed achieve very high levels of hedging effectiveness. For the base case results with a planning horizon of one year, which are shown in Panel A, hedging effectiveness ranges from 82.84% for oil and \( q = 0.5 \) up to 97.15% for the exchange rate and \( q = 0.1 \), indicating that, for all markets and all specifications of payment date uncertainty, substantial reductions in standard deviation can be attained. Looking more closely at the different commodities, it can be seen that the effectiveness is highest for gold and then copper and lowest for oil. This finding is consistent with the importance of convenience yield risk in hedging. Hedging effectiveness is lowest for the commodity with the highest convenience yield risk (oil) and highest for the commodity with the lowest convenience yield risk (gold). Moreover, the effectiveness of the foreign currency hedge, where convenience yields do not affect forward prices, is consistently higher than for the commodities.

As a robustness check, the firm’s planning horizon has been changed from one year to two months and two years, respectively. The results are presented in Panels B

\(^{25}\) The original measure uses variance instead of standard deviation (see Johnson (1960)).
and C of Table 1. In the former case, hedging effectiveness is higher across the board for the commodities with the ordering of the markets in terms of hedging effectiveness remaining unchanged. For a planning horizon of two years, the hedging effectiveness is lower than in the base case, but the ordering of the markets again stays the same. For the exchange rate, both shorter and longer horizons reduce hedging effectiveness. This result can be explained as follows: Hedging effectiveness is negatively affected by the variation in the interest rate differential between the United States and the Euro zone. If the hedge horizon lengthens (shortens), there are two effects: First, the longer (shorter) the time between date 0 and date 1, the more likely are larger (smaller) interest rate changes within this period. Second, if the hedge horizon shortens, the relevant interest rates are the one-month rates instead of the six-month rates. However, shorter-term interest rates show stronger variation over time than longer-term interest rates. For a two-month planning horizon, this second effect overcompensates for the first effect, whereas, for the two-year planning horizon, the first one is more important then the second.

### 3.2.2. Improvements on Alternative Strategies

The optimal strategy allows for high levels of hedging effectiveness; however, whether it really reduces risk compared to simple heuristic alternatives has yet to be evaluated. Table 2 presents the percentage reduction in standard deviations if one switches from either the long-term, the rollover, or the deferred strategy to the optimal strategy. Panel A shows the base case results for a planning horizon of one year. Several conclusions can be drawn: First, risk can often be reduced substantially by adopting the optimal strategy for all markets and all values of $q$. Second, the highest levels of risk reduction can be achieved when replacing the deferred strategy with the optimal strategy. This finding is also consistent over all $q$s and all markets. The third result is that the values for long-term and rollover strategies are exact opposites with respect to the value of $q$. Assuming the probability of a sale after the
first period is 10%, it is clear that a rollover hedge is less well suited to reduce exposure and therefore a change to the optimal strategy will yield great improvements. Should the likelihood of a sale at \( t = 1 \) be 90%, however, a long-term strategy is less appropriate and, consequently, the potential for risk reduction via the optimal strategy is higher.

\[ \text{[Insert Table 2 about here]} \]

In many instances, there will be no telling when a sale is more likely to occur. Therefore, \( q = 0.5 \) represents a good starting point for the evaluation. Even in this scenario of highest possible uncertainty, the standard deviation of the hedged position can be reduced by about 6% for the USD–Euro exchange rate and by as much as 25% for oil when the current strategy is a long-term or a rollover hedge. If the deferred hedge has been the strategy of choice, reductions of up to 92% for gold are made possible by the optimal hedge. The ordering of the markets is not as clear as with the hedging effectiveness but depends on the alternative strategy. After crude oil, copper exhibits the most potential for volatility reduction for the long-term and rollover strategies, whereas a greater reduction can be achieved for gold when switching from the deferred strategy.

With probabilities other than 0.5, even greater reductions are possible for firms following a deferred strategy and, depending on whether the chance of a sale at \( t = 1 \) increases or decreases, for the other two strategies as well. When hedging exposure to gold price risk with the likelihood of a sale at \( t = 1 \) being 0.9, for example, the standard deviation can be reduced by 95% for the deferred hedge and by 50% for the long-term hedge. For the deferred hedge, the values are only slightly lower for the other markets and, with respect to the long-term strategy, volatility reductions of up to 65% are possible for crude oil, for example.

Again, we change the planning horizon for robustness checks. In a setup with two one-month periods, as reported in Panel B, similar improvements can be achieved
by opting for the optimal strategy. In the case of the deferred hedge, volatility can be reduced even more than in the previous setting. For oil and copper, this also holds for the long-term and the rollover strategies. The values for the exchange rate hedge are almost unchanged for those strategies. As the results for Panel C show, extending the planning horizon to two years does not alter the general result that the optimal hedge delivers significant improvements over heuristic alternatives that are employed in practice. The reduction in volatility is somewhat lower in this setting, but, with the expanded time horizon, this is not surprising. Therefore, all in all, the optimal hedge as derived in Corollary 1 can enhance corporate risk management when uncertainty about the payment date is a factor, and it can do so in different markets, as well as for different time horizons.

3.3. Results: Dependent Payment Date and Price

We now extend our analysis to allow for both customized non-linear payoff structures and dependence between the payment date and the price to assess the performance of the optimal strategy in Proposition 2. We compare it with a rollover and a long-term strategy, as well as the unconditional linear strategy given by Corollary 1 and the optimal linear strategy from Proposition 1. We exclude the comparison with both the no-hedge scenario and a deferred hedge, since a sophisticated non-linear optimal strategy is more likely to be considered by risk management officers with greater experience, who will already have some sort of policy in place.

[ Insert Table 3 about here ]

Table 3, Panel A, presents the percentage reductions in standard deviation for the base case (one-year planning horizon). It can be seen that the non-linear hedge is indeed superior to all the other strategies, especially the long-term and rollover strategies. If the dependence level is high (i.e., $\beta = 8$ in Equation (14)), volatility
reductions can be as high as 60% for crude oil, 50% for copper, and 25% for gold when the reference point is the long-term strategy.

Compared to the unconditional strategy, the exotic hedge can yield improvements between 7% for gold and 28% for crude oil. Even when the optimal strategy replaces the optimal linear hedge, it can lead to reductions in volatility of up to 20% in the case of crude oil. Generally speaking, the higher the dependence between price and payment date, the better the non-linear strategy fares compared to the alternatives. While improvements for the currency hedge are somewhat lower, our findings still suggest there could be a market for customized over-the-counter derivatives tailored specifically to such exposures.

As in the previous section, we alter the hedging period to two months and two years, the results of which are presented in Panels B and C of Table 3, respectively. For the two-month horizon, the overall improvements achieved by the optimal strategy are less pronounced than in the base case. This is especially true for the exchange rate and gold, where the optimal linear strategy and the unconditional linear strategy perform almost as well. For copper and crude oil, the improvements are greater, especially when the dependence between prices and payment date is high. Then, the optimal strategy can reduce volatility by more than one-third compared to the long-term hedge.

In the two-year case, the exotic hedge can lead to a substantial outperformance of the optimal linear strategy when dependence is high. For gold, the reduction in volatility for that case amounts to about 6% and reaches 16% for copper and even 22% for crude oil. When compared to the unconditional linear strategy, improvements can be as high as 44% and, compared to the long-term strategy, up to 68%. Even when dependence is only moderate, the optimal strategy can reduce volatility by up to 57%. Interestingly, the results for copper and oil also show that a rollover strategy could perform even better than considering the correct unconditional probability of an early payment and using the unconditional linear strategy if there is
dependence between payment date and price risk.

All in all, this illustrates two important results. First, a hedge as prescribed by the optimal strategy can substantially reduce volatility in profits, especially when payment date and price are strongly dependent and when the hedge horizon increases. The intuition behind this result is the following: The stronger the dependence, the stronger the non-linear relation between profits and prices becomes and the less accurate is a linear approximation. Moreover, the longer the hedge horizon, the more likely are large price moves falling in a region where the linear approximation is particularly bad. Second, for shorter periods and with moderate dependence, the linear optimal strategy works quite well, so it may not be necessary to always consider dependence between payment date and price in one's hedging policy, particularly for the USD–Euro exchange rate. This latter result can help explain the findings of Gay et al. (2002) and Huang et al. (2007), in which a large part of surveyed firms use only linear derivatives to hedge currency exposure.

4. Conclusions

Uncertainty about the timing of cash flows to be hedged is a challenge for the design of hedging strategies that is prevalent in many actual situations. In particular, it raises interesting questions about the instrument choice that we study in this paper. Using a discrete-time model, we derive dynamic hedging strategies that minimize the variance of profits in two settings: The first setting uses futures contracts of different maturities and the second allows for (exotic) options written on these futures. A linear hedging strategy is shown to be optimal if price and payment date are independent. In this case, only knowledge of the probabilities with respect to the timing of future cash flows is required to determine the optimal maturity mix. A non-linear hedging strategy can reduce profit variance even more when payment date and price are dependent random variables. The key element of this strategy is the conditional probability that the payment occurs at a specific date, given all
available price information. Implementation of this strategy requires exotic options written on multiple prices, referring to futures with different maturities.

Our empirical study quantifies the performance of the derived hedging strategies for three commodities and the USD–Euro exchange rate over a variety of scenarios. We show that the optimal hedge can deliver high levels of effectiveness in different markets. For a firm with a planning horizon of one year, a minimum reduction in standard deviation of 82% can be achieved in commodity markets, regardless of the probability of an early sale. For the USD–Euro exchange rate, the minimum value is even 97%. For the shorter horizon of two months, the minimum reduction for crude oil increases to about 89% while it is still 94% for the currency hedge. Even for a two-year horizon, variance reductions between 79% and 96% are possible for the four markets and different probabilities of an early sale.

We also show that the optimal hedge substantially outperforms several heuristic hedging strategies, regardless of the length of the planning horizon and the probability of an early sale. Especially when compared to a deferred hedge, that is, a hedging strategy that starts hedging only if exposure is known exactly, an optimal hedge can reduce the standard deviation of total profits by up to 95% in the one-year setting. In a two-month timeframe, even greater risk reductions are possible. Very substantial improvements over rollover or long-term hedging strategies are similarly possible but depend on the probabilities $q$ for an early sale. For $q = 0.5$, the improvements range from 6% for the exchange rate hedge to 25% for crude oil. We generally find that the higher the convenience yield risk of a commodity, the greater the resulting improvements of the optimal hedging strategy over the long-term and rollover strategies. Overall, our results indicate that the derived optimal strategies present a superior way to hedge price risk when payment dates are uncertain.

Finally, we ask how closely hedging with linear instruments resembles the usage of non-linear exotic derivatives with an optimal payoff structure. For currency hedges with short hedge horizons and moderate dependence between price risk and
the uncertainty about the payment date, the optimal linear strategy can suffice. Given that managers generally seem to be more familiar with linear derivatives and liquid futures markets are often available, hedging with linear instruments is a reasonable choice. For strong dependence between price and payment date and longer hedge horizons, however, a linear approximation of the non-linear relation between profits and prices is less accurate and tailor-made non-linear derivatives can lead to significant improvements in terms of risk reduction.
Appendix

Proof of Proposition 1: The minimization problem (7) is a dynamic one that has to be solved recursively. Therefore, the determination of the optimal hedging policy starts at date 1.

Date 1: According to Assumption 1, the use of derivatives does not change expected profits, which implies that variance minimization is equivalent to minimizing $E_0[\Pi^2]$. Therefore, the problem to be solved at date 1 can be written

$$\min_{h_{1,2}} E_1 [\Pi^2],$$

(16)

This optimization problem leads to the following first-order condition, which is also sufficient for a minimum:

$$\frac{\partial E_1 [\Pi^2]}{\partial h_{1,2}} = E_1 [2 \Pi (P_2 - F_{1,2})] = 0$$

$$\Leftrightarrow \text{Cov}_1 [\Pi, (P_2 - F_{1,2})] = 0.$$  

(17)

The equivalence of the two conditions in Equation (17) follows from the assumption that futures earn zero expected profits. Note that previous hedging decisions taken at date 0 enter the optimum condition in Equation (17) via the profit $\Pi$. Explicitly, this can be seen from Equation (6).

According to Equation (17), we require that profit be uncorrelated with the payoff of the futures contract. Correlation refers to the joint conditional distribution of profit and futures, conditional on information available at date 1. There is certainly no correlation if profit (conditional on information available at date 1) is not stochastic. Note that at date 1 the payment date is known. If $I = 1$, the sale has occurred and there is no price exposure. If $I = 0$, operating profits are risky solely because of the uncertain price $P_2$. By choosing $h_{1,2} = -Q$, we completely eliminate
this price risk. In conclusion, buying \(h_{1,2} = -Q(1-I)\) futures contracts leads to a certain profit from the perspective of date 1, which means that the optimality condition is fulfilled.

**Date 0:** Given the optimal futures position at date 1, the firm’s profit becomes

\[
\Pi^* = P_1 Q I (1 + r) + F_{1,2} Q (1 - I) - C
\]

\[
+ (P_1 - F_{0,1})(1 + r) h_{0,1} + (F_{1,2} - F_{0,2}) h_{0,2}
\]

and the optimization problem can be written

\[
\min_{h_{0,1}, h_{0,2}} E_0[\Pi^{*2}].
\]

The minimization problem (19) leads to the following first-order conditions:

\[
\frac{\partial E_0[\Pi^{*2}]}{\partial h_{0,1}} = E_0[2 \Pi^* (P_1 - F_{0,1})(1 + r)] = 0
\]

\[\Leftrightarrow\]

\[
Cov_0[\Pi^*, (P_1 - F_{0,1})(1 + r)] = 0,
\]

\[
\frac{\partial E_0[\Pi^{*2}]}{\partial h_{0,2}} = E_0[2 \Pi^* (F_{1,2} - F_{0,2})] = 0
\]

\[\Leftrightarrow\]

\[
Cov_0[\Pi^*, (F_{1,2} - F_{0,2})] = 0,
\]

Solutions to the first-order conditions also satisfy the second-order conditions for a minimum. The solution is unique if the price changes of the two relevant futures contracts are not perfectly correlated, that is, if neither contract is redundant.

Let us define \(\bar{h}_{0,1} := h_{0,1}/Q, \bar{h}_{0,2} := h_{0,2}/Q,\) and \(\bar{P}_1 = P_1(1 + r).\) Then, using the more compact matrix notation, the optimality conditions (20) and (21) can be
written

\[
\begin{pmatrix}
\text{Cov}_0 \left[ I(\bar{P}_1 - F_{1,2}) + \bar{P}_1 \bar{h}_{0,1} + F_{1,2}(1 + \bar{h}_{0,2}), \bar{P}_1 \right]
\text{Cov}_0 \left[ I(\bar{P}_1 - F_{1,2}) + \bar{P}_1 \bar{h}_{0,1} + F_{1,2}(1 + \bar{h}_{0,2}), F_{1,2} \right]
\end{pmatrix}
= \begin{pmatrix}
0 \\
0
\end{pmatrix}
\] (22)

\[
\Leftrightarrow \begin{pmatrix}
\text{Cov}_0 \left[ I(\bar{P}_1 - F_{1,2}), \bar{P}_1 \right]
\text{Cov}_0 \left[ I(\bar{P}_1 - F_{1,2}), F_{1,2} \right]
\end{pmatrix}
= -\begin{pmatrix}
\text{Var}_P & \text{Cov}_{P,F} \\
\text{Cov}_{P,F} & \text{Var}_F
\end{pmatrix}
\begin{pmatrix}
\bar{h}_{0,1} \\
1 + \bar{h}_{0,2}
\end{pmatrix}
\] (23)

Splitting the covariance on the left-hand side of Equation (23) into two parts, we obtain

\[
\begin{pmatrix}
\text{Cov}_0 \left[ I\bar{P}_1, \bar{P}_1 \right] \\
\text{Cov}_0 \left[ I\bar{P}_1, F_{1,2} \right]
\end{pmatrix}
- \begin{pmatrix}
\text{Cov}_0 \left[ IF_{1,2}, \bar{P}_1 \right] \\
\text{Cov}_0 \left[ IF_{1,2}, F_{1,2} \right]
\end{pmatrix}
= \begin{pmatrix}
\text{Var}_P & \text{Cov}_{P,F} \\
\text{Cov}_{P,F} & \text{Var}_F
\end{pmatrix}
\begin{pmatrix}
F_{0,1}(1 + r) \\
F_{0,2}
\end{pmatrix}
\] (24)

Note that for three random variables \(X, Y, Z\), \(\text{Cov}(XY,Z) = E[X]\text{Cov}(Y,Z) + E[Y]\text{Cov}(X,Z) + E[(X - E(X))(Y - E(Y))(Z - E(Z))]\) (Bohrnstedt and Goldberger (1969)). Based on this result, the difference between the two covariance terms above equals

\[
\begin{pmatrix}
\text{Var}_P & \text{Cov}_{P,F} \\
\text{Cov}_{P,F} & \text{Var}_F
\end{pmatrix}
\begin{pmatrix}
q \\
-q
\end{pmatrix}
+ \begin{pmatrix}
\text{Cov}_{P,I} & -\text{Cov}_{P,I} \\
\text{Cov}_{F,I} & -\text{Cov}_{F,I}
\end{pmatrix}
\begin{pmatrix}
F_{0,1}(1 + r) \\
F_{0,2}
\end{pmatrix}
\] (25)

\[
+ \begin{pmatrix}
\text{CoSk}_{P,P,I} - \text{CoSk}_{P,F,I} \\
\text{CoSk}_{P,F,I} - \text{CoSk}_{F,F,I}
\end{pmatrix},
\]

where we use the fact that \(E_0[I] = q, E_0[\bar{P}_1] = F_{0,1}(1 + r)\) and \(E_0[F_{1,2}] = F_{0,2}\). Inserting the expressions from Equation (25) into Equation (23) and solving for \(h_{0,1}\) and \(h_{0,2}\) produces the result. □

**Proof of Proposition 2:** To prove the proposition, we have to solve the dynamic minimization problem (11). It has to be solved recursively, starting at date 1.
**Date 1:** At date 1, we consider the Lagrange function $E_1 \left[ \Pi^2 - \lambda_2 g_2(P_2) \right]$ and solve the following functional optimization problem:

$$\min_{g_2(P_2)} E_1 \left[ \Pi^2 - \lambda_2 g_2(P_2) \right]. \quad (26)$$

The first-order conditions for this problem, which are also sufficient for a minimum, can be written

$$\frac{\partial E_1[\Pi^2-\lambda_2 g_2(P_2)]}{\partial g_2(P_2)} = E_1[2 \Pi - \lambda_2 | P_2] = 0$$

$$\iff E_1[\Pi|P_2] = \lambda_2/2, \quad \forall P_2 \in (0, \infty). \quad (27)$$

Since uncertainty about the payment date is no longer present at date 1 and $P_2$ is the only remaining random variable, we can drop the expectation in Equation (27). Optimality requires that the profit be constant for all realizations of $P_2$. As we have seen in the proof of Proposition 1, selling $Q(1-I)$ forward contracts leads to a non-stochastic profit. Therefore, the corresponding payoff function fulfills the optimality conditions and $g_2^*(P_2) = -Q(1-I)(P_2 - F_{1,2})$.

**Date 0:** Given the optimal hedging policy at date 1, the firm’s profit becomes

$$\Pi^* = P_1 Q I (1 + r) + F_{1,2} Q (1 - I) - C + g_1(P_1, F_{1,2})(1 + r). \quad (28)$$

At date 0, the relevant Lagrange function is $E_0[\Pi^{*2} - \lambda_1 g_1(P_1, F_{1,2})]$ and we have to solve the following functional optimization problem:

$$\min_{g_1(P_1, F_{1,2})} E_0 \left[ \Pi^{*2} - \lambda_1 g_1(P_1, F_{1,2}) \right]. \quad (29)$$
The first-order conditions also provide sufficient conditions for this problem:

\[
\frac{\partial E_0[\Pi^* - \lambda_1 g_1(P_1, F_{1,2})]}{\partial g_1(P_1, F_{1,2})} = E_0 \left[ 2 \Pi^* - \lambda_1 |P_1, F_{1,2} \right] = 0
\]

\[\Leftrightarrow\]

\[
E_0 \left[ \Pi^* |P_1, F_{1,2} \right] = \frac{\lambda_1}{2}, \text{ } \forall \text{ } P_1 \in (0, \infty), \text{ } F_{1,2} \in (0, \infty).
\]

Equation (30) states that the expected profit, conditional on \(P_1\) and \(F_{1,2}\), must be the same for all values of \(P_1\) and \(F_{1,2}\). Note that the expectations in Equation (30) are taken with respect to the random variable \(I\). The conditions of Equation (30) are fulfilled if 

\[
g_1(P_1, F_{1,2}) = -Q \left[ q_{|P_1, F_{1,2}} P_1 + (1 - q_{|P_1, F_{1,2}}) F_{1,2} (1 + r)^{-1} - k \right],
\]

where \(q_{|P_1, F_{1,2}}\) denotes the conditional probability that a sale occurs at date 1, that is, the expectation of \(I|P_1, F_{1,2}\). The constant \(k\) ensures that the expected payoff (with respect to \(P_1\) and \(F_{1,2}\)) of the derivatives contract equals zero. Therefore, the specific value of \(k\) depends on the joint distribution of \(P_1\) and \(F_{1,2}\) and the dependence structure of price and payment date. \(\Box\)
References


This figure shows the optimal payoff functions of exotic derivatives contracts written on the spot price $P_1$ and the futures price $F_{1,2}$ for different levels of dependence between price and payment date. The top part of the figure refers to the case of independent risks ($\beta = 0$), the middle part to a moderate level of dependence ($\beta = 4$), and the bottom part to strong dependence ($\beta = 8$).
This table shows the hedging effectiveness of the optimal strategy according to Equation (15) for the gold, copper, crude oil, and FX markets. The uncertainty of the payment date is characterized by the probability $q$ of a sale taking place at $t = 1$. This probability does not depend on the price at $t = 1$. Panel A of the table refers to the base case planning horizon of one year. Panels B and C refer to planning horizons of two months and two years, respectively.

### Panel A: One-year horizon

| $q$ | Gold | Copper | Crude Oil | $$/€$
---|---|---|---|---|
| 0.1 | 96.87 | 94.43 | 90.85 | 97.15 |
| 0.3 | 94.91 | 91.20 | 85.23 | 97.04 |
| 0.5 | 94.03 | 89.65 | 82.84 | 97.02 |
| 0.7 | 94.05 | 89.68 | 83.10 | 97.02 |
| 0.9 | 95.68 | 92.51 | 87.95 | 97.09 |

### Panel B: Two-month horizon

| $q$ | Gold | Copper | Crude Oil | $$/€$
---|---|---|---|---|
| 0.1 | 97.84 | 95.37 | 94.11 | 96.69 |
| 0.3 | 96.55 | 92.47 | 90.46 | 94.65 |
| 0.5 | 96.04 | 91.18 | 88.68 | 93.76 |
| 0.7 | 96.15 | 91.24 | 88.89 | 93.82 |
| 0.9 | 97.31 | 93.68 | 92.09 | 95.56 |

### Panel C: Two-year horizon

| $q$ | Gold | Copper | Crude Oil | $$/€$
---|---|---|---|---|
| 0.1 | 96.42 | 92.87 | 88.42 | 88.53 |
| 0.3 | 94.11 | 88.39 | 81.58 | 81.79 |
| 0.5 | 93.01 | 86.33 | 78.98 | 79.16 |
| 0.7 | 92.89 | 86.34 | 79.75 | 83.66 |
| 0.9 | 94.68 | 90.04 | 85.98 | 85.73 |
### TABLE 2
Hedging improvements when payment date and price are independent

**Panel A: One-year horizon**

<table>
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Panel B: Two-month horizon

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Copper

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Crude Oil

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Panel C: Two-year horizon

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Copper

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Crude Oil

This table shows the reduction in standard deviation when switching from the respective strategies to the optimal hedge given in Corollary 1. Values are in percent and q denotes the likelihood of a sale taking place at the end of the first period. Panel A refers to the base case planning horizon of one year. Panels B and C refer to planning horizons of two months and two years, respectively.
TABLE 3  
Hedging improvements when payment date and price are dependent

Panel A: One-year horizon

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Panel B: Two-month horizon

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Panel C: Two-year horizon

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| Moderate| 13.10     | 24.37     | 57.39     | 19.60    |
| High    | 22.24     | 37.07     | 67.45     | 26.92    |

This table shows the reduction in standard deviation when switching from the respective strategies to the optimal hedge under if payment date and price are dependent. The optimal strategy is given in Proposition 2. Dependence levels are modeled according to Equation (14) with $\beta = 4$ for moderate and $\beta = 8$ for high dependence. Values are in percent. Panel A refers to the base case planning horizon of one year. Panels B and C refer to planning horizons of two months and two years, respectively.