Sinn und Bedeutung 2014
Georg-August-Universität Göttingen, 15.9.2014

At most at last

Doris Penka
Universität Konstanz
doris.penka@uni-konstanz.de

1. Superlative modifiers and ignorance inferences

The meaning of the superlative modifiers *at least* and *at most* provides interesting challenges for semantic and pragmatic analysis:

- In most contexts, *at least* and *at most* imply speaker ignorance, i.e. they convey that the speaker isn't sure about the precise value (see Geurts & Nouwen 2007, Nouwen 2010):

(1) John had **at least** five beers last night.

(2) #I have **at least**/ **at most** three children.

- In certain environments, the implication of speaker uncertainty vanishes. In particular, it can be suppressed in certain combinations of *at least*/*at most* with modals, which give rise to a so called authoritative reading (see Geurts & Nouwen 2007, Büring 2008):

▪ necessity modal + *at least*:

(3) a. The paper is **required** to be **at least** 10 pages long.
'10 pages is the minimally required length of the paper'
[---------- authoritative reading
10pp

b. [I don't know exactly. But I think:] The paper is **required** to be **at least** 10 pages long.
'According to what the speaker knows, the minimally required length might be 10 pages or it might be more.'
[/////////---------- speaker insecurity reading
10pp

▪ possibility modal + *at most*:

(4) a. The paper is **allowed** to be **at most** 10 pages long.
'10 pages is the maximally allowed length of the paper'
---------] authoritative reading
10pp

b. [I don't know exactly. But I think:] The paper is **allowed** to be **at most** 10 pages long.
'According to what the speaker knows, the maximally allowed length might be 10 pages or less.'
----------/////////] speaker insecurity reading
10pp

The ignorance implications of *at least* and *at most* and their interaction with modals are currently subject to a lot of work in semantics and pragmatics:

- Geurts & Nouwen (2007) hard-wire speaker ignorance into the lexical entry of superlative modifiers.
- Nouwen (2010) attributes speaker ignorance to a covert epistemic modal, which rescues a structure that would otherwise be ruled out for economy reasons.
- Büring (2008), Cummins & Katsos (2010), Schwarz (2011, 2013), Kennedy (2013) derive ignorance inferences triggered by *at least* and *at most* as quantity implicatures arising via Gricean reasoning.
- Coppock & Brochhagen (2013) argue that superlative modifiers denote sets of alternatives (possibilities) in the framework of inquisitive semantics and attribute speaker ignorance to a Maxim of Interactive Sincerity.
- Cohen & Krifka (2014) treat superlative modifiers as modifiers of speech acts.

None of these analyses fully accounts for the interaction of *at least* and *at most* and modals.

Plot of the talk:

- Pragmatic analyses that derive ignorance inferences of superlative modifiers as quantity implicatures are conceptually attractive and empirically well motivated.
- But while they successfully account for the interaction of *at least* with modals, they fail for the interaction of *at most* with modals.
- If *at most* is decomposed into an antonymizing operator and *at least*, a pragmatic account correctly predicts that *at most* gives rise to an authoritative reading in combination with possibility modals.
- The decompositional analysis also accounts for further patterns of interaction between *at most* and other operators.
- This further supports the idea that negative antonyms are generally decomposed in the syntax.

2. Ignorance inferences of *at least* and *at most* as quantity implicatures

2.1 Basic idea

Büring (2008), Cummins & Katsos (2010), Schwarz (2011, 2013) and Kennedy (2013) propose that the ignorance inferences triggered by *at least* and *at most* arise as quantity implicatures.

Following Schwarz (2011), ignorance inferences of *at least* and *at most* can be derived in the Neo-Gricean approach underlying scalar implicatures. Ignorance inferences are derived in the same way as ignorance implications for disjunction under Sauerland's (2004) analysis.

The essential ingredients of the analysis are the following:

- *at least* and *at most* are degree operators expressing non-strict comparison:

(5) a. $[\![\text{at least}]\!] = \lambda d_d.\ \lambda D_{dt}.\ \max(D) \geq d$
b. $[\![\text{at most}]\!] = \lambda d_d.\ \lambda D_{dt}.\ \max(D) \leq d$

- Utterances with *at least n* or *at most n* trigger scalar alternatives in which
  - *n* is substituted by other numerals/ measure phrases
  - *at least* and *at most* are substituted by *exactly*

(6) $[\![\text{exactly}]\!] = \lambda d_d.\ \lambda D_{dt}.\ \max(D) = d$

2.2 Unembedded occurrences

Ignorance inferences for unembedded occurrences of *at least* and *at most* arise because the more informative (logically stronger) **alternatives are symmetric** (i.e. they cannot simultaneously be false while the assertion is true) and thus block the derivation of scalar implicatures and lead to ignorance inferences.

- ***at least***

(7) The paper is **at least** 10 pages long.
max{d: long(p,d)} ≥ 10pp

(8) Scalar alternatives to (7):
The paper is *NumMod n* pages long. where *NumMod* ∈ { *at least, exactly, at most* }
n ∈ { …, 9, 10, 11, …

(9) Relevant stronger scalar alternatives: [1] symmetric!
a. The paper is **exactly** 10 pages long. *at least* ➡ *exactly*
max{d: long(p,d)} = 10pp
b. The paper is at least **11** pages long. *10pp* ➡ *11pp*
max{d: long(p,d)} ≥ 11pp <==>*
max{d: long(p,d)} > 10pp

(10) Ignorance implicatures generated:
a. P max{d: long(p,d)} = 10pp & P ¬max{d: long(p,d)} = 10pp
b. P max{d: long(p,d)} > 10pp & P ¬max{d: long(p,d)} > 10pp
'The speaker doesn't know whether the paper is exactly 10pp long or whether the paper is more than 10pp long.'

- ***at most***

(11) The paper is **at most** 10 pages long.
max{d: long(p,d)} ≤ 10pp

(12) Relevant stronger scalar alternatives: symmetric!
a. The paper is **exactly** 10 pages long. *at most* ➡ *exactly*
max{d: long(p,d)} = 10pp
b. The paper is at most **9** pages long. *10pp* ➡ *9pp*
max{d: long(p,d)} ≤ 9pp <==>*
max{d: long(p,d)} < 10pp

(13) Ignorance implicatures generated:
a. P max{d: long(p,d)} = 10pp & P ¬max{d: long(p,d)} = 10pp
b. P max{d: long(p,d)} < 10pp & P ¬max{d: long(p,d)} < 10pp
'The speaker doesn't know whether the paper is exactly 10pp long or whether the paper is less than 10pp long.'

2.3 Interaction with necessity modals

Because superlative modifiers are analyzed as degree operators, two different scope orders are

[1] I consider only those scalar alternatives that assymetrically entail the assertion and where the number is closest to the modified numeral. The latter might be problematic; see Mayr (2013) and Schwarz (2013).
* For simplicity, I assume that the relevant scale is discrete, i.e. that only full-page lengths are considered.

possible when they interact with modals.

- □ > *at least*:

If superlative modifiers are interpreted in the scope of a necessity modal, the more informative **alternatives are not symmetric (**they can be false while the assertion is true, namely if the permissible paper length corresponds to a range), and thus scalar implicatures rather than ignorance inferences are generated.

(14) The paper is **required** to be **at least** 10 pages long.

(15) a. required [[at least 10pp] λd [the paper be d long]] □ > *at least*
b. □ max{d: long(p,d)} ≥ 10pp
'In all the acceptable worlds, the length of the paper is 10pp or more.'

(16) Relevant stronger scalar alternatives: not symmetric!
a. □ max{d: long(p,d)} = 10pp *at least* ➡ *exactly*
'In all the acceptable worlds, the length of the paper is exactly 10pp.'
b. □ max{d: long(p,d)} ≥ 11pp <==>* *10pp* ➡ *11pp*
□ max{d: long(p,d)} > 10pp
'In all the acceptable worlds, the paper is longer than 10pp.'

(17) Scalar implicatures generated:
a. K¬□max{d: long(p,d)} = 10pp
b. K ¬□max{d: long(p,d)} > 10pp
'The speaker is sure that the paper doesn't have to be exactly 10pp long and that the paper doesn't have to be more than 10pp long.'

Taken together, the assertion and the scalar implicatures express that the permissible paper lengths correspond to a range of values whose lower bound is 10pp. This corresponds to the authoritative reading.

(18) [---------- authoritative reading
10pp

- *at least* > □:

If *at least* takes wide scope, the speaker insecurity reading results. Although the scope order *at least* > □ is truth-conditionally equivalent to □ > *at least*, the pragmatic reasoning is different and the former gives rise to ignorance inferences rather than scalar implicatures (this is because narrow scope of *exactly n* in the scalar alternatives leads to symmetric alternatives):

(19) a. [at least 10pp] λd [required [the paper be d long]] *at least* > □
b. max{d: □ long(p,d)} ≥ 10pp
'The minimally required length of the paper is 10pp or more.'

(20) Relevant stronger scalar alternatives: symmetric!
a. max{d: □ long(p,d)} = 10pp *at least* ➡ *exactly*
'The minimally required length of the paper is exactly 10pp.'
b. max{d: □ long(p,d)} ≥ 11pp <==>* max{d: □ long(p,d)} > 10pp *10pp* ➡ *11pp*
'The minimally required length of the paper is more than 10pp.'

(21) Ignorance implicatures generated:
a. P max{d: □ long(p,d)} = 10pp & P ¬ max{d: □ long(p,d)} = 10pp
b. P max{d: □ long(p,d)} > 10pp & P ¬ max{d: □ long(p,d)} > 10pp
'The speaker doesn't know whether the minimally required length of the paper is exactly 10pp or whether the minimally required length of the paper is more than 10pp.'

(22) [/////////---------- speaker insecurity reading
10pp

Parallel for *at most*:

(23) The paper is **required** to be **at most** 10 pages long.

- <u>□ > *at most*</u>:

(24) a. required [[at most 10pp] λd [the paper be d long]] □ > *at most*
b. □ max{d: long(p,d)} ≤ 10pp
'In all the acceptable worlds, the length of the paper is 10pp or less.'

(25) Relevant stronger scalar alternatives: not symmetric!
a. □ max{d: long(p,d)} = 10pp *at most* ➡ *exactly*
'In all the acceptable worlds, the length of the paper is exactly 10pp.'
b. □ max{d: long(p,d)} ≤ 9pp <==>* □ max{d: long(p,d)} < 10pp *10pp* ➡ *9pp*
'In all the acceptable worlds, the paper is shorter than 10pp.'

(26) Scalar implicatures generated:
a. K¬□max{d: long(p,d)} = 10pp
b. K¬□max{d: long(p,d)} < 10pp
'The speaker is sure that the paper doesn't have to be exactly 10pp long and that the paper doesn't have to be less than 10pp long.'

(27) ----------] authoritative reading
10pp

- <u>*at most* > □</u>:

(28) a. [at most 10pp] λd [required [the paper be d long]] *at most* > □
b. max{d: □ long(p,d)} ≤ 10pp
'The minimally required length of the paper is 10pp or less.'

(29) Relevant stronger scalar alternatives: symmetric!
a. max{d: □ long(p,d)} = 10pp *at most* ➡ *exactly*
'The minimally required length of the paper is exactly 10pp.'
b. max{d: □ long(p,d)} ≤ 9pp <==>* *10pp* ➡ *9pp*
max{d: □ long(p,d)} < 10pp
'The minimally required length of the paper is less than 10pp.'

(30) Ignorance implicatures generated:
a. P max{d: □ long(p,d)} = 10pp & P ¬ max{d: □ long(p,d)} = 10pp
b. P max{d: □ long(p,d)} < 10pp & P ¬ max{d: □ long(p,d)} < 10pp
'The speaker doesn't know whether the minimally required length of the paper is exactly 10pp or whether the minimally required length of the paper is less than 10pp.'

(31) ////////]---------- speaker insecurity reading
10pp

- In general, ignorance inferences are obviated if a superlative modifier is interpreted in the scope of an operator that breaks symmetry. This also accounts for other cases of ignorance obviation, e.g. universal quantifiers (Schwarz, 2011) and generics (Nouwen 2010).

(32) **Every** paper on the reading list is **at least** 10 pages long.
~> Some paper(s) on the reading list are exactly 10 pages long.
~> Some paper(s) on the reading list are more than 10 pages long

2.4 Interaction with possibility modals

When superlative modifiers are combined with possibility modals, the more informative **alternatives of both scope orders are symmetric** and thus ignorance inferences are generated.

(33) The paper is **allowed** to be **at least** 10 pages long.

- ◇ > at least:

(34) a. allowed [[at least 10pp] λd [the paper be d long]] ◇ > *at least*
b. ◇ max{d: long(p,d)} ≥10pp
'There is an acceptable world where the length of the paper is 10pp or more.'

(35) Relevant stronger scalar alternatives: symmetric!
a. ◇max{d: long(p,d)} = 10pp *at least* ➡ *exactly*
'There is an acceptable world where the length of the paper is exactly 10pp.'
b. ◇max{d: long(p,d)} ≥ 11pp <==>* *10pp* ➡ *11pp*
◇max{d: long(p,d)} > 10pp
'There is an acceptable world where the length of the paper is more than 10pp.'

(36) Ignorance inferences generated:
a. P ◇ max{d: long(p,d)} = 10pp & P ¬◇max{d: long(p,d)} = 10pp
b. P ◇ max{d: long(p,d)} > 10pp & P ¬◇ max{d: long(p,d)} > 10pp
'The speaker doesn't know whether the paper is allowed to be exactly 10pp long and whether the paper is allowed to be more than 10pp long.'

According to these ignorance inferences the speaker doesn't even know whether 10pp is a permissible option, i.e. for all he knows the minimum length might be above 10pp. Intuitively, this seems wrong. Therefore the ignorance inferences are too strong. This reading might not be detectable because there is another reading with sensible ignorance inferences.

- *at least* > ◇:

(37) a. [at least 10pp] λd [allowed [the paper be d long]] *at least* > ◇
b. max{d: ◇long(p,d)} ≥ 10pp
'The maximally allowed length of the paper is 10pp or more.'

(38) Relevant stronger scalar alternatives: symmetric!
a. max{d: ◇long(p,d)} = 10pp *at least* ➡ *exactly*
'The maximally allowed length of the paper is exactly 10pp.'
b. max{d: ◇long(p,d)} ≥ 11pp <==>* max{d: ◇long(p,d)} > 10pp *10pp* ➡ *11pp*
'The maximally allowed length of the paper is more than 10pp.'

These alternatives are symmetric and thus lead to ignorance inferences according to which the

speaker isn't sure whether the maximally allowed length of the paper is exactly 10pp or whether the maximally allowed length is above10pp.

(39) Ignorance implicatures generated:
a. P max{d: ◇long(p,d)} = 10pp & P ¬max{d: ◇long(p,d)} = 10pp
b. P max{d: ◇long(p,d)} > 10pp & P ¬max{d: ◇long(p,d)} > 10pp
'The speaker doesn't know whether the maximally allowed length of the paper is exactly 10pp or whether the maximally allowed length of the paper is more than 10pp.'

(40) ----------[///////// speaker insecurity reading
10pp

Parallel for *at most*:

(41) The paper is **allowed** to be **at most** 10 pages long.

- ◇ > *at most*:

(42) a. allowed [[at most 10pp] λd [the paper be d long]] ◇ > *at most*
b. ◇ max{d: long(p,d)} ≤ 10pp
'There is an acceptable world where the length of the paper is 10pp or less.'

(43) Relevant stronger scalar alternatives: symmetric!
a. ◇max{d: long(p,d)} = 10pp *at most* ➡ *exactly*
'There is an acceptable world where the length of the paper is exactly 10pp.'
b. ◇max{d: long(p,d)} ≤ 9 pp <==>* *10pp* ➡ *9pp*
◇max{d: long(p,d)} < 10pp
'There is an acceptable world where the length of the paper is less than 10pp.'

(44) Ignorance inferences generated:
a. P ◇ max{d: long(p,d)} = 10pp & P ¬◇max{d: long(p,d)} = 10pp
b. P ◇ max{d: long(p,d)} < 10pp & P ¬◇ max{d: long(p,d)} < 10pp
'The speaker doesn't know whether the paper is allowed to be exactly 10pp long and whether the paper is allowed to be less than 10pp long.'

Again, these ignorance inferences are too strong.

- *at most* > ◇:

(45) a. [at most 10pp] λd [allowed [the paper be d long]] *at most* > ◇
c. max{d: ◇long(p,d)} ≤ 10pp
'The maximally allowed length of the paper is 10pp or less.'

(46) Relevant stronger scalar alternatives: symmetric!
a. max{d: ◇long(p,d)} = 10pp *at most* ➡ *exactly*
'The maximally allowed length of the paper is exactly 10pp.'
b. max{d: ◇long(p,d)} ≤ 9pp <==>* *10pp* ➡ *9pp*
max{d: ◇long(p,d)} < 10pp
'The maximally allowed length of the paper is less than 10pp.'

(47) Ignorance implicatures generated:
a. P max{d: ◇long(p,d)} = 10pp & P ¬max{d: ◇long(p,d)} = 10pp
b. P max{d: ◇long(p,d)} < 10pp & P ¬max{d: ◇long(p,d)} < 10pp
'The speaker doesn't know whether the maximally allowed length of the paper is exactly 10pp or whether the maximally allowed length of the paper is less than 10pp.'

(48) ----------/////////] speaker insecurity reading
10pp

2.5 Summary of predictions of the Neo-Gricean account

The Neo-Gricean analysis predicts that ignorance inferences are obviated in combination with a necessity modal, but not in combination with a possibility modal.

The following patter is predicted:

- □ > *at least/ at most*: authoritative reading
- ◇ > *at least/ at most*: strong ignorance reading
- *at least/ at most* > □/◇: speaker insecurity reading

This correctly accounts for the readings observed for *at least*.

(49) a. The paper is **required** to be **at least** 10 pages long.
'10 pages is the minimally required length of the paper'
[---------- authoritative reading
10pp

b. [I don't know exactly. But I think:] The paper is **required** to be **at least** 10 pages long.
'According to what the speaker knows, the minimally required length might be 10 pages or it might be more.'
[/////////---------- speaker insecurity reading
10pp

(50) The paper is **allowed** to be **at least** 10 pages long.
'According to what the speaker knows, the maximally allowed length might be 10 pages or it might be more.'
----------[///////// speaker insecurity reading only
10pp

But not for *at most*: The authoritative reading is available for the combination of *at most* with possibility modals (see McNabb & Penka 2014a, 2014b for experimental evidence).

(51) a. The paper is **allowed** to be **at most** 10 pages long.
'10 pages is the maximally allowed length of the paper'
---------] authoritative reading
10pp

b. [I don't know exactly. But I think:] The paper is **allowed** to be **at most** 10 pages long.
'According to what the speaker knows, the maximally allowed length might be 10 pages or less.'
----------/////////] speaker insecurity reading
10pp

Experimental evidence (McNabb & Penka 2014a, 2014b) moreover suggests that the authoritative reading isn't as readily available for *at most* in combination with a necessity modal as it is in combination with a possibility modal, contrary to the predictions.

3. A decompositional analysis of *at most*

3.1 Basic idea

- I propose that *at most n* is decomposed into an antonymizing operator ANT and *at least n*:

(52) at most n = $[[n\text{-ANT}]_{d(dt)}\text{-at least}]_{(dt)t}$

This follows the idea that negative antonyms are generally decomposed in the syntax into an antonymizing operator and the corresponding positive antonym (Heim 2006, 2008; Büring 2007; Alxatib 2013).

(53) $[\![\text{at least}]\!] = \lambda d_d.\ \lambda D_{dt}.\ \max(D) \geq d$

- What is the semantics of the antonymizing operator ANT?

Heim's (2006) *little*:

(54) $[\![\text{little}]\!] = \lambda d_d.\ \lambda D_{dt}.\ \neg D(d)$

But to account for the meaning of sentences like (55) a revision of Heim's definition of *little* is needed, in which the degree contributed by the first argument isn't negated (see also Beck 2012):

(55) [Mary only weighs 40 kg.] Sue weighs that little too.

(56) Meaning derived for (55) with Heim's definition of *little* in (54):
a. [that little] λd [Sue weighs d-much]
b. ¬[WEIGHT(s) ≥ 40kg] =
WEIGHT(s) < 40kg
'Sue weighs less than 40 kg.'

(57) $[\![\text{little}_2]\!] = \lambda d_d.\ \lambda D_{dt}.\ \forall d'>d:\ \neg D(d')$

This derives the correct meaning for the sentence in (55):

(58) a. [that little_2] λd [Sue weighs d-much]
b. ∀d' > 40kg: ¬[WEIGHT(s) ≥ d'] =
¬[WEIGHT(s) > 40kg]
'Sue doesn't weigh more than 40 kg.'

As noted by Beck (2012), this renders *little* or the antonymizing operator ANT equivalent to the straightforward definition of *at most* from (5b) above!

(59) $[\![\text{ANT}]\!] = \lambda d_d.\ \lambda D_{dt}.\ \forall d'>d:\ \neg D(d')$

(8b) $[\![\text{at most}]\!] = \lambda d_d.\ \lambda D_{dt}.\ \max(D) \leq d$

- Assumptions about scalar alternatives (compatible with Katzir's (2007) and Fox & Katzir's (2011) definition of alternatives): Alternatives are generated by
  - substituting numerals/ measure phrases by each other
  - substituting *at least* by *exactly* (Schwarz 2011)
  - deleting ANT (see Alxatib 2013)
  - substituting modals by each other

3.2 Unembedded occurrences of *at most*

Ignorance inferences arising with *at most* are generated in the same way as in the Neo-Gricean

analysis: the more informative **alternatives are symmetric** and thus lead to ignorance inferences.

(60) The paper is **at most** 10 pages long.

(61) a. ANT-10 λd_2 [at least-d_2 λd_1 [the paper is d_1-long]]
b. $\forall$d' > 10pp: $\neg$ [max{d: long(p,d)} $\geq$ d] <==>
$\neg$ max{d: long(p,d)} > 10pp

(62) Scalar alternatives:
The paper is *Pol NumMod n* pages long. where *Pol* $\in$ { ANT, $\varnothing$ }
NumMod $\in$ { *at least, exactly* }
n $\in$ { …, 9, 10, 11, …}

(63) Relevant stronger scalar alternatives: symmetric!
a. The paper is **exactly**10 pages long. ~~ANT~~, *at least* ➡ *exactly*
max{d: long(p,d)} = 10pp
b. The paper is at most **9** pages long. *10pp* ➡ *9pp*
$\neg$ max{d: long(p,d)} > 9pp <==>* max{d: long(p,d)} < 10pp

(64) Ignorance inferences:
a. P max{d: long(p,d)} = 10pp & P $\neg$max{d: long(p,d)} = 10pp
b. P max{d: long(p,d)} < 10pp & P $\neg$max{d: long(p,d)} < 10pp
'The speaker isn't sure whether the paper is exactly 10pp long or whether the paper is less than 10pp long.'

3.3 Interaction of *at most* with possibility modals

When *at most* is combined with a modal, we need to consider three different scope orders:

(65) The paper is **allowed** to be **at most** 10 pages long.

(66) a. allowed [ANT-10pp λd_2 [at least-d_2 λd_1 [the paper be d_1-long]]] $\Diamond$ > ANT > at least
b. ANT-10pp λd_2 [at least-d_2 λd_1 [allowed [the paper be d_1-long]]] ANT > at least > $\Diamond$
c. ANT-10pp λd_2 [allowed [at least-d_2 λd_1 [the paper be d_1-long]]] ANT > $\Diamond$ > at least

The decompositional analysis makes available an LF where ANT takes wide and *at least* takes narrow scope wrt. the modal. The alternatives are **not symmetric** and thus the Neo-Gricean approach derives scalar implicatures resulting in the authoritative reading.

- ANT > $\Diamond$ > *at least*:

(67) a. ANT-10pp λd_2 [allowed [at least-d_2 λd_1 [the paper be d_1-long]]]
b. $\forall$d' > 10pp: $\neg\Diamond$ max{d: long(p,d)} $\geq$ d <==>
$\neg\Diamond$ max{d: long(p,d)} > 10pp
'There is no acceptable world where the length of the paper is above10pp.'

(68) Scalar alternatives:
The paper is *Pol Mod NumMod n* pages long. where *Pol* $\in$ { ANT, $\varnothing$ }
Mod $\in$ { *allowed, required* }
NumMod $\in$ {*at least, exactly* }
n $\in$ { …, 9, 10, 11, …}

(69) Relevant stronger scalar alternatives: not symmetric!
a. ¬◇ max{d: long(p,d)} > 9pp *10pp ➡ 9pp*
b. □ max{d: long(p,d)} = 10pp ~~ANT~~, ◇ ➡ □, *at least ➡ exactly*

(70) Scalar implicatures generated:
a. K ◇ max{d: long(p,d)} > 9pp
b. K ¬□ max{d: long(p,d)} = 10pp
'The speaker is sure that the paper is allowed to be more than 9pp long and he is sure that the paper doesn't have to be exactly 10pp long.'

Taken together, the assertion and the scalar implicatures express that the permissible paper lengths correspond to a range of values whose upper bound is 10pp. This corresponds to the authoritative reading.

(71) ----------] authoritative reading
10pp

For the other two LFs where both ANT and *at least* take scope over or under the possibility modal, the same readings are derived as for non-decomposed *at most* under the analysis of Schwarz (2011):

- ◇ > ANT > *at least*

(72) a. allowed [ANT-10pp λd_2 [at least-d_2 λd_1 [the paper be d_1-long]]]
b. ◇∀d' > 10pp: ¬[max{d: long(p,d)} ≥ d'] <==>
◇ max{d: long(p,d)} ≤ 10pp
'There is an acceptable world where the length of the paper is 10pp or less.'

(73) Relevant scalar alternatives: a. and b. symmetric, c. and d. not
a. ◇max{d: long(p,d)} ≤ 9pp <==>* *10pp ➡ 9pp*
◇max{d: long(p,d)} < 10pp
b. ◇max{d: long(p,d)} = 10pp ~~ANT~~, *at least ➡ exactly*
c. □ max{d: long(p,d)} ≤ 10pp ◇ ➡ □
d. □ max{d: long(p,d)} = 10pp ~~ANT~~, ◇ ➡ □, *at least ➡ exactly*

(74) Ignorance Inferences:
a. P◇ max{d: long(p,d)} < 10pp & P ¬◇ max{d: long(p,d)} < 10pp
b. P◇ max{d: long(p,d)} = 10pp & P ¬◇ max{d: long(p,d)} = 10pp
'The speaker doesn't know whether the paper is allowed to be exactly 10pp long and whether the paper is allowed to be less than 10pp long.'

(75) Scalar implicatures:
a. K¬□ max{d: long(p,d)} ≤ 10pp
b. K ¬□ max{d: long(p,d)} = 10pp
'The speaker is sure that the paper doesn't have to be exactly 10pp long and that the paper doesn't have to be 10pp long or shorter.'

The ignorance inferences are too strong and the scalar implicatures are innocuous.

- ANT > *at least* > ◇

(76) a. ANT-10pp λd_2 [at least-d_2 λd_1 [allowed [the paper be d_1-long]]]

b. ∀d' > 10pp: ¬[max{d: ◇long(p,d)} ≥ d'] <==>
¬[max{d: ◇long(p,d)} > 10pp] <==>
max{d: ◇long(p,d)} ≤ 10pp
'The maximally allowed length of the paper is 10pp or less.'

(77) Relevant scalar alternatives:
a. max{d: ◇long(p,d)} ≤ 9pp <==>* *10pp ➡ 9pp*
max{d: ◇long(p,d)} < 10pp
b. max{d: ◇long(p,d)} = 10 pp ~~ANT~~, *at least ➡ exactly*

(78) Ignorance inferences generated:
a. P max{d: ◇long(p,d)} = 10pp & P ¬max{d: ◇long(p,d)} = 10pp
b. P max{d: ◇long(p,d)} < 10pp & P ¬max{d: ◇long(p,d)} < 10pp
'The speaker isn't sure whether the maximally allowed length is exactly 10pp or less than 10pp.'

(79) ----------/////////] speaker insecurity reading
10pp

3.4 Interaction of *at most* with necessity modals

When *at most* is combined with a necessity modal, we again have to consider three scope orders.

(80) The paper is **required** to be **at most** 10 pages long.

(81) a. required [ANT-10 λd$_2$ [at least-d$_2$ λd$_1$ [the paper be d$_1$-long]]] □ > ANT > at least
b. ANT-10 λd$_2$ [at least-d$_2$ λd$_1$ [required [the paper be d$_1$-long]]] ANT > at least > □
c. ANT-10 λd$_2$ [required [at least-d$_2$ λd$_1$ [the paper be d$_1$-long]]] ANT > □ > at least

Again, the readings derived for the two LFs where both ANT and *at least* take scope over or under the modal are equivalent to the ones for non-decomposed *at most*. The LF where a necessity modal takes scope in between ANT and *at least* leads to strong ignorance inferences, and the reading is thus probably not detectable.

- ANT > □ > *at least*:

(82) a. ANT-10pp λ_{d2} [required [at least-$_{d2}$ λ_{d1} [the paper be $_{d1}$-long]]]
b. ∀d' > 10pp: ¬□ max{d: long(p,d)} ≥ d <==>
¬□ max{d: long(p,d)} > 10pp
'The paper isn't required to be more than 10pp long.'

(83) Relevant stronger scalar alternatives: a. and b. symmetric, c. not
a. ¬□ max{d: long(p,d)} > 9pp *10pp ➡ 9pp*
b. ◇ max{d: long(p,d)} = 10pp ~~ANT~~, □ ➡ ◇, *at least ➡ exactly*
c. ¬◇ [max{d: long(p,d)} > 10pp] □ ➡ ◇

(84) Ignorance inferences generated:
a. P ¬□ max{d: long(p,d)} > 9pp & P □ max{d: long(p,d)} > 9pp
b. P ◇ max{d: long(p,d)} = 10pp & P ¬◇ max{d: long(p,d)} = 10pp
'The speaker isn't sure whether the paper is required to be longer than 9pp and he isn't sure whether the paper is allowed to be exactly 10pp long.'

(85) Scalar implicatures generated:
a. K ◇ max{d: long(p,d)} > 10pp
'The speaker is sure that the paper is allowed to be longer than 10pp.'

- ANT > *at least* > □:

(86) a. ANT-10pp λd_2 [at least-d_2 λd_1 [required [the paper be d_1-long]]]
b. ∀d' > 10pp: ¬[max{d: □long(p,d)} ≥ d'] <==>
¬[max{d: □ long(p,d)} > 10pp] <==>
max{d: □ long(p,d)} ≤ 10pp
'The minimally required length is 10 pp or less.'

(87) Relevant scalar alternatives:
a. max{d: □ long(p,d)} ≤ 9pp <==>* *10pp ➡ 9pp*
max{d: □ long(p,d)} < 10pp
b. max{d: □ long(p,d)} = 10pp ~~ANT~~, *at least ➡ exactly*

(88) Ignorance inferences generated:
a. P max{d: □ long(p,d)} < 10pp & P ¬max{d: □ long(p,d)} < 10pp
b. P max{d: □ long(p,d)} = 10pp & P ¬max{d: □ long(p,d)} = 10pp
'The speaker isn't sure whether the minimally required length is exactly 10pp or less than 10pp.'

(89) ////////]---------- speaker insecurity reading
10pp

- □ > ANT > *at least*

(90) a. required [ANT-10pp λd_2 [at least-d_2 λd_1 [the paper be d_1-long]]]
b. □∀d' > 10pp: ¬[max{d: long(p,d)} ≥ d'] <==>
□ ¬[max{d: long(p,d)} > 10pp] <==>
□ max{d: long(p,d)} ≤ 10pp
'In every acceptable world, the length of the paper is 10pp or less.'

(91) Relevant scalar alternatives:
a. □ max{d: long(p,d)} ≤ 9pp <==>* *10pp ➡ 9pp*
□ max{d: long(p,d)} < 10pp
b. □ max{d: long(p,d)} = 10pp ~~ANT~~, *at least ➡ exactly*

(92) Scalar implicatures generated:
a. K ¬□ max{d: long(p,d)} < 10pp
b. K ¬□ max{d: long(p,d)} = 10pp
'The speaker is sure that the paper doesn't have to be less than 10pp long and that the paper doesn't have to be exactly 10pp long.'

(93) ----------]$_{10pp}$ authoritative reading

- The analysis thus inherits from the Neo-Gricean accounts the prediction that an authoritative reading is available for the combination of *at most* with a necessity modal.
- If *at most* is embedded in a finite clause under a necessity modal, the authoritative reading is readily available:

(94) a. [I am looking for suggestions for a dorm room microwave for my son.]
The college **requires** that it be **at most** 1 cu feet in volume and **at most** 800 Watts.
'1 cu feet is the maximally allowed volume and 800 Watts is the maximally allowed power.'
http://www.bogleheads.org/forum/viewtopic.php?f=11&t=100975 (accessed 30 June 2014).
b. This algorithm **requires** that variables be used **at most** once.
'The maximally allowed number of variable uses is one.'
http://homepages.inf.ed.ac.uk/wadler/papers/oncetech/oncetech.ps (accessed 30 June 2014)

- The interaction of *at most* with necessity modals can be accounted for by stipulating that ANT takes wide scope over a modal if it can do so.

4. Further predictions of the analysis

4.1 Scope trapping

- Deriving the authoritative reading of *at most* plus possibility modal from an LF where ANT takes wide scope also correctly predicts that the authoritative reading isn't available if movement out of the scope of the modal is blocked for independent reasons, in particular when *at most* is embedded in a finite clause.

(95) It is **permitted** that the paper is **at most** 10 pages long.
*'10 pages is the maximally allowed length of the paper.'

This also constitutes evidence against anttributing obviation of ignorance inferences under possibility modals to a Free Choice effect (contra Coppock & Brochhagen 2013).

4.2 Interaction with other operators

In their interaction with non-modal existential and universal operators, *at least* and *at most* behave parallel. Both *at least* and *at most* obviate ignorance inferences under universal quantifiers, but give rise to strong ignorance inferences under (singular) existential quantifiers.

(96) **Every** paper on the reading list is **at least** 10 pages long.
~> Some paper(s) on the reading list are exactly 10 pages long.
~> Some paper(s) on the reading list are more than 10 pages long.

(97) **Every** paper on the reading list is **at most** 10 pages long.
~> Some paper(s) on the reading list are exactly 10 pages long.
~> Some paper(s) on the reading list are less than 10 pages long.

(98) **Some** paper on the reading list is **at least** 10 pages long.
~> The speaker doesn't know whether some paper is exactly 10 pages long.
~> The speaker doesn't know whether some paper is more than 10 pages long.

(99) **Some** paper on the reading list is **at most** 10 pages long.
~> The speaker doesn't know whether some paper is exactly 10 pages long.
~> The speaker doesn't know whether some paper is less than 10 pages long.

Under the decompositional analysis this difference in the interaction with modals and quantifiers follows from the Heim-Kennedy-generalization (Heim 2000), according to which degree operators can QR over modals, but not quantifiers. Thus ANT as well as *at least* obligatorily take scope under quantifiers.

The following patter is predicted:

- □/ ∀ > (ANT >) *at least*: authoritative reading
- ◇/ ∃ > (ANT >) *at least*: strong ignorance reading
- (ANT >) *at least* > □/◇: speaker insecurity reading
- (ANT >) *at least* > ∀/∃: ---
- ANT > ∀/∃ > *at least*: ---
- ANT > ◇ > *at least*: authoritative reading
- ANT > □ > *at least*: strong ignorance reading

5. Conclusions

- If the Neo-Gricean approach is supplemented with the assumption that *at most* is decomposed into an antonymizing operator and *at least,* it successfully accounts for "the bewildering interaction of *at least* and *at most* with modals and other operators" (Schwarz 2013: 193).
- This lends further support to the idea that negative antonyms are generally decomposed in the syntax (Büring, 2007).
- But the analysis also raises the question what triggers this decomposition:
  - For the antonym pair *at least*/ *at most* it seems that semantic rather than morphological properties are decisive.
  - It is the downward monotonic modifier *at most* that involves the antonymizing operator, not upward monotonic *at least,* which is morphologically based on *little.*

References

Alxatib, Sam. 2013. 'Only' and association with negative quantifiers. Doctoral Dissertation, MIT.
Beck, Sigrid. 2012. DegP Scope Revisited. *Natural Language Semantics* 20, 227-272.
Büring, Daniel. 2007. *More* or *less*. Proceedings of Chicago Linguistics Society.
Büring, Daniel. 2008. The least *at least* can do. Proceedings of WCCFL 26, 114-120.
Cohen, Ariel & Manfred Krifka. 2014. Superlative quantifiers and meta-speech acts. *Linguistics and Philosophy* 37. 41-90.
Coppock, Elizabeth & Thomas Brochhagen. 2013b. Raising and resolving issues with scalar modifiers. *Semantics & Pragmatics* 6. 1–57.
Cummins, Chris & Napoleon Katsos. 2010. Comparative and superlative quantifiers: Pragmatic effects of comparison type. *Journal of Semantics* 27(3). 271 –305.
Fox, Danny. 2007. Free choice disjunction and the theory of scalar implicatures, in U. Sauerland

& P. Stateva (ed.): Presupposition and implicature in compositional semantics, 71–120. Palgrave MacMillan
Fox, Dannny, and Roni Katzir. 2011. On the characterization of alternatives. *Natural Language Semantics* 19:87–107.
Gazdar, Gerald. 1979. *Pragmatics: Implicature, Presupposition, and Logical Form*, Academic Press, NY.
Geurts, Bart and Rick Nouwen. 2007. 'At least' et al.: The semantics of scalar modifiers. *Language* 83(3), 533–559.
Heim, Irene. 2000. Degree Operators and Scope. *Proceedings of SALT X*, ed. by Brendan Jackson and Tanya Matthews, 40–64. Ithaca, NY: CLC Publications.
Heim, Irene. 2006. *Little*. *Proceedings of SALT 16*, ed. by M. Gibson and J. Howell. Ithaca, NY: CLC Publications.
Heim, Irene. 2008. Decomposing Antonyms? In Grønn, Atle (ed.): *Proceedings of SuB12*, 212–225. Oslo: ILOS.
Katzir, Roni. 2007. Structurally-defined alternatives. *Linguistics and Philosophy* 30:669–690.
Kennedy, Christopher. 2013. A "de-Fregean semantics for modified and unmodified numerals. Ms., University of Chicago.
Kratzer, Angelika, and Junko Shimoyama. 2002. Indeterminate phrases: The view from Japanese. In *The Proceedings of the Third Tokyo Conference on Psycholinguistics*, ed. Otsu, Yokio, 1–25. Tokyo: Hituzi Syobo.
Mayr, Clemens. 2013. Implicatures of modified numerals. *From grammar to meaning: the spontaneous logicality of language*, ed. by Ivano Caponigro and Carlo Cecchetto, 139-171. Cambridge University Press.
McNabb, Yaron & Doris Penka. 2014a. The interpretation of superlative modifiers and deontic modals: An experimental investigation. In Proceedings of Sinn und Bedeutung 18, ed. by U. Etxeberria, A. Fălăuş, A. Irurtzun, B. Leferman, 271-288. Vitoria-Gasteiz.
McNabb, Yaron & Doris Penka. 2014b. The processing cost of interpreting superlative modifiers and modals. In J. Degen, M. Franke, and N. Goodman (Eds.), Proceedings of the Formal & Experimental Pragmatics Workshop, 29-35. Tübingen.
Nouwen, Rick. 2010. Two kinds of modified numerals. *Semantics and Pragmatics* 3: 1–41.
Nouwen, Rick. (to appear). Modified Numerals: The Epistemic Effect. In *Epistemic Indefinites*, ed. by Luis Alonso-Ovalle, L. and Paula Menendez-Benito. Oxford University Press.
Sauerland, Uli. 2004. Scalar implicatures in complex sentences. *Linguistics and Philosophy*, 27:367-391.
Schwarz, Bernhard. 2011. Remarks on Class B numeral modifiers. Handout of a talk at the workshop Indefinites and Beyond, Universität Göttingen, November 19, 2011.
Schwarz, Bernhard. 2013. *At least* and Quantity Implicature: Choices and Consequences. Proceedings of the 19th Amsterdam Colloquium, ed. by Maria Aloni, Michael Franke and Floris Roelofsen.