At most at last

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SUMMARY In this presentation, I show that the interaction of *at most* and modals, which is a puzzle for otherwise successful pragmatic accounts of ignorance inferences arising with superlative modifiers, can be explained by decomposing *at most* into an antonymizing operator and *at least.*

BACKGROUND There is currently a lot of work on the semantics and pragmatics of the superlative modifiers *at least* and *at most*. Analyses have to account for two facts in particular: First, unembedded uses of superlative modifiers as in (1a) convey speaker ignorance (Nouwen 2010). Second, certain combinations of *at least* and *at most* with modals allow readings without ignorance inferences (Geurts & Nouwen 2007): Whereas *at least* in combination with a necessity modal leads to an authoritative reading (1b), in which the modified numeral specifies the lower bound of a range of permissible values, *at most* leads to an authoritative reading specifying the upper bound in combination with a possibility modal (1c).

(1) a. The paper is at least/ at most 10 pages long.
↝ The speaker isn't sure about the exact length of the paper.
b. The paper is required to be at least 10 pages long.
'10 pages is the minimally required length of the paper.'
c. The paper is allowed to be at most 10 pages long.
'10 pages is the maximally allowed length of the paper.'

Pragmatic accounts (Büring 2008; Schwarz 2013; Kennedy 2013) derive the ignorance inferences of superlative modifiers as quantity implicatures in a neo-Gricean framework, in which ignorance inferences rather than scalar implicatures are generated in case the stronger scalar alternatives are symmetric, i.e. cannot simultaneously be false while the assertion is true. Schwarz (2013) and Kennedy (2013) analyse superlative modifiers as degree operators with the semantics in (2) and argue that they obligatorily trigger scalar alternatives, which in the analysis of Schwarz (2013) correspond to the cross-product of substituting superlative modifiers by *exactly* and the modified numeral by other numerals.

(2) a $[\![\text{at least}]\!] = \lambda d_d.\ \lambda D_{dt}.\ \max(D) \geq d$ b. $[\![\text{at most}]\!] = \lambda d_d.\ \lambda D_{dt}.\ \max(D) \leq d$

In the case of (1a), the stronger alternatives *The paper is exactly 10 pages long* and *The paper is at least 11 pages long* are symmetric and thus lead to ignorance inferences according to which the speaker is unsure whether the paper is exactly 10pp or longer than 10pp. Pragmatic accounts also explain that ignorance inferences are absent in downward entailing contexts and that the modified numeral doesn't trigger scalar implicatures. They predict that ignorance inferences are obviated whenever symmetry is broken, which is the case in the scope of necessity modals, but not in combination with possibility modals. Pragmatic analyses thus successfully account for the interaction of *at least* with modals, but cannot explain the authoritative reading of *at most* arising in connection with possibility modals.

PROPOSAL I propose that *at most* is decomposed into an antonymizing operator ANT, with the semantics in (3b) also proposed by Beck (2012), and its positive counterpart *at least* as defined in (2a) above:

(3) a *at most* n = $[[\text{ANT-n}]_{d(dt)}\text{-}\textit{at least}]_{(dt)t}$
b. $[\![\text{ANT}]\!] = \lambda d_d.\ \lambda D_{dt}.\ \forall d' > d: \neg D(d')$

Adopting a structural complexity approach where scalar alternatives are derived by lexical

substitution and deletion (Katzir 2007), the alternatives considered for utterances with at most are alternatives generated by (i) substituting the modified numeral; (ii) substituting at least by exactly (Schwarz 2013); (iii) deleting ant (Alxatib 2013); (iv) substituting modals. For unembedded occurrences of at most as in (4a), the result of this analysis is equivalent to assuming the lexical entry in (2b), and the truth conditions in (4b) are derived. Similarly as for at least, the stronger scalar alternatives The paper is at most 9 pages long and The paper is exactly 10 pages long are symmetric and thus lead to ignorance implications, according to which the speaker considers both a length of exactly 10pp and less than 10pp possible.

(4) a. The paper is at most 10 pages long.
b. ANT-10pp λd_2 [at least-d_2 λd_1 [the paper is d_1-long]]
c. $\forall d' > 10pp$: $\neg$[max{d: long(p,d)} > d'] $\Leftrightarrow$ max{d: long(p,d)} $\leq$ 10pp

Crucially, when *at most* is combined with a possibility modal, the decompositional analysis makes available a scope order obviating ignorance inferences. While $\Diamond$ > ANT > *at least* and ANT > *at least* > $\Diamond$ both lead to ignorance inferences, ANT > $\Diamond$ > *at least* leads to scalar implicatures rather than ignorance implications, cf. (5). In this case, the stronger scalar alternatives in (5d-i) (derived by substituting *10* by *9*) and (5d-ii) (derived by deleting ANT and substituting *allowed* by *required* and *at least* by *exactly*) aren't symmetric and thus serve as basis for the scalar implicatures in (5e), which together with the assertion entail that the paper is allowed to be exactly 10pp long and it is allowed to be shorter than 10 pp, but not longer than 10pp. This successfully accounts for the authoritative reading of (5a) specifying10pp as the maximally allowed length.

(5) a. The paper is allowed to be at most 10 pages long.
b. ANT-10pp λd_2 [allowed [at least-d_2 λd_1 [the paper be d_1-long]]]
c. $\forall d' > 10pp$: $\neg\Diamond$[max{d: long(p,d)} $\geq$ d'] $\Leftrightarrow$ $\neg\Diamond$[max{d': long(p,d')} > 10pp]
d. (relevant) scalar alternatives: i. $\neg\Diamond$[max{d: long(p,d)} > 9pp]
ii. $\Box$ [max{d: long(p,d)} = 10pp]
e. scalar implicatures: i. $\Diamond$[max{d: long(p,d)} > 9pp]
ii. $\neg\Box$ [max{d: long(p,d)} = 10pp]

Deriving the authoritative reading of (5a) from an LF where ANT takes wide scope also correctly predicts that the authoritative reading isn't available if movement out of the scope of the modal is blocked for independent reasons, e.g. when *at most* is embedded in a finite clause (6a). It also explains that ignorance inferences are obviated under existential modals, but not existential quantifiers as in (6b), since quantifiers are known to block movement of degree operators.

(6) a. It is allowed that the paper is at most 10 pages long.
*'10 pages is the maximally allowed length of the paper.'
b. Some paper on the reading list is at most 10 pages long.
*'10 pages is the length of the longest paper on the reading list.'

References: **Alxatib**, S. 2013. 'Only' and association with negative quantifiers. PhD dissertation, MIT. **Beck**, S. 2012. DegP Scope Revisited. *Natural Language Semantics* 20, 227-272. **Büring**, 2008. The least *at least* can do. *WCCFL* 26, 114-120. **Geurts**, B. & R. **Nouwen**. 2007. 'At least' et al.: The semantics of scalar modifiers. *Language* 83(3):533–559. **Katzir**, R. 2007. Structurally-defined alternatives. *Linguistics and Philosophy* 30:669–690. **Kennedy**, C. 2013. A "de-Fregean semantics for modified and unmodified numerals. Ms., University of Chicago. **Nouwen**, R. 2010. Two kinds of modified numerals. *Semantics and Pragmatics* 3:1–41. **Schwarz**, B. 2013. *At least* and Quantity Implicature: Choices and Consequences. 19th *Amsterdam Colloquium*.