# Coalition Governments and Political Rents 

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#### Abstract

We analyze the impact of coalition governments on the ability of political competition to eliminate political parties' rents. We show that political parties earn positive rents due to the stochastic nature of the coalition government formation procedure which softens political competition. A further problem is that the middle party which will be a part of all potential coalition governments cannot be disciplined at all. Another finding is that parties differentiate in terms of policy programs in order to earn more rents, since voters are ready to pay more rents with differentiated policy choices of parties.

Arguing that proportional rule systems lead more often to coalition governments than plurality rule systems, we predict higher political rents in proportional rule systems. Our prediction is confirmed empirically.


Keywords: electoral competition, rent-seeking political parties, coalition governments
JEL Classication: D72, D73, D78

## 1 Introduction

We analyze whether political competition is able to prevent political parties' rent-seeking behavior. Kunicova and Rose-Ackerman (2005) defines corrupt rent-seeking of elected officials as "the misuse of public office for private financial gain by an elected official" such as embezzlement of public funds. Rent-seeking can be also legal, such as high salaries and legal privileges. The common wisdom is that political parties would not seek to have rents since otherwise they are not able to attract votes and so to be in power (see e.g. Wittman (1989)). More specifically, in this paper, we question this common wisdom in the context of coalition governments. We believe that it is an important question given the prevalence of coalition governments. In 13 western european countries over the period 1945-1999, 69 per cent of governments were coalitions (238 of 343 cases). (Muller and Strom (2000))

From our viewpoint, the crucial property of coalition governments is that their formation process is stochastic. In other words, even after the election results are known, nobody, including parties, can be sure about which parties will take part of the coalition government. Precisely for this reason, there is a large literature investigating coalition formation. This literature includes among others Muller and Strom (2000) and Laver and Schofield (1998). This uncertainty creates a lack of political accountability as Diermeier and Merlo (2004) also recognize by saying "if changes in electoral outcomes do not lead to corresponding changes in government composition, voters may perceive this as a lack of control over their elected representatives." Due to the uncertain nature of the coalition government formation process, although a party decreasing her rent level increases her vote share and her probability of being in the coalition government, she is never completely sure of being in the government. Her rent level is the result of a trade-off between her probability of being in government and her rent level she will enjoy if she ends up in the government. In other terms, this stochastic nature of the government formation process softens political competition and results in positive rents in equilibrium. Moreover, if a party (the middle party in our model) expects to be a member of all potential coalition governments ${ }^{1}$, she has clearly no incentive to decrease her rent level. Therefore, we conclude that political competition in the context of coalition governments is not able to eliminate rents.

Moreover, by endogenizing the ideal policy choices of political parties, we find diverging policy choices. In addition to a well known centripetal force making parties going more

[^1]towards the median policy choice in order to increase their vote shares, we also have a centrifugal force making parties going towards the extreme policy choices. Once the parties realize that they are able to have positive rent levels as a result of softened political competition, they are looking for a way to increase them: They differentiate so that voters accept to vote for them despite higher rent levels.

In our setup, a two-party competition leading to a single-party government would result in parties choosing the median policy and extracting no rent. In this context, there is no uncertainty about the government formation and a party can secure victory by choosing a more favorable policy and rent level to voters than other parties. Consequently, this harsh competition would be able to eliminate rents totally and would result in parties choosing the median policy.

Given that proportional rule systems lead to multi-party competition and to coalition governments more often than plurality rule systems ${ }^{2}$, our empirical prediction is that proportional rule systems lead to more political rents than plurality rule systems. Kunicova and Rose-Ackerman (2005) and Persson, Tabellini and Trebbi (2003) study empirically the effect of electoral rule on corruption and find that proportional rule systems lead to more corruption than plurality rule systems. As they argue, although corruption is more comprehensive than political rents, it is a relevant proxy. However, their theoretical explanations differ from ours. While agreeing with their explications that we discuss in the section on related literature, we are proposing an additional one, the stochastic nature of coalition formation process.

An important uncertainty about coalition government formation is which party will be given the role of proposing a government, namely the role of formateur. Hence, we choose to model the uncertainty in coalition formation by the use of the probabilistic formateur selection rule. This rule states that a party is chosen as formateur with probability equal to her vote share. Diermeier and Merlo (2004) show that the probabilistic selection rule fits better the data than a deterministic selection rule stating that the party with the highest vote share will be chosen as formateur. Indeed, for more than $40 \%$ of cases they cover (313 observations in 11 multi-party democracies over the period 1945-1997), the largest party is not chosen as formateur.

In a variant of our model, we change the government formation procedure. We replace the probabilistic selection rule by the fixed selection rule. According to this rule, the party with the highest vote share is selected as formateur to form the coalition government. The main implication of this rule is that the party with most votes is able to form the coalition

[^2]government. Hence, the government formation process is not probabilistic. Consequently, parties, except the middle party which will be a part of any coalition government, compete harshly to be attract the highest vote share, and as shown in the corresponding section, this competition results in these parties choosing the median policy and earning no rent. However, the middle party chooses again the highest possible rent level as in the case of the probabilistic selection rule, since she will be a part of any coalition government anyway. This extension confirms our intuition that our main results, namely positive rents and diverging policy platforms, are due to the stochastic nature of the probabilistic selection rule, except the middle party's rent. Hence, we suggest that the problem caused by coalition governments can be partly fixed by defining a clear and deterministic government formation process. However, today's situation is better reflected by the probabilistic selection rule than by the fixed selection rule, as discussed above.

Our model consists in three political parties competing to be part of a two-party coalition government. In other words, we take the necessity of coalition governments as given. Our objective is not to explain the existence of coalition governments, but rather to see their effects on political competition. Voters have uniformly distributed policy preferences on a unidimensional policy parameter and they dislike paying rents. First, political parties announce their policy programs. Parties do not have any intrinsic policy preferences. Hence, in the terminology of the political economy literature, they are office-motivated. Second, they announce their rent levels they will extract when in government. Hence, before the elections, voters know perfectly the policy programs and the rent levels of the three political parties. As remarked by Myerson (1993), this is the most difficult case for parties to extract rents, since rents are public information. If parties are able to extract rents even in this case, they will be able a fortiori in other and possibly more realistic cases too. Third, voters vote strategically. Finally a party is chosen to be a formateur, according to the probabilistic selection rule defined above, and a coalition government is formed. The two coalition parties extract their announced rent levels and decide on the final policy. The out-of-government party does not extract any rent.

When a voter decides on his vote, he has two concerns: he wants the party closer with his policy orientation to be in the government and he wants to avoid paying rents. When a political party other than the middle party decides on her rent level, she realizes that a higher rent level reduces her vote share and so her probability of being in the government, but if she is in the government eventually, she will enjoy a higher rent level. Hence, she has a trade-off between her probability of extracting rents and the amount of rents extracted if in government. The crucial element is that even with a very low level of rents, she cannot
be sure to be in the government, due to the probabilistic nature of the coalition government formation. Hence, she chooses an optimal level of rents according to her above trade-off. At the beginning of the game, when she decides on her policy program, she realizes that a policy program closer to that of the median voter increases her probability of being in the government. However, she has also a reason to choose a policy program farther away from those of the median voter. When she differentiates her policy program, she obtains a better trade-off at the following rent decision stage. The reason is that with more differentiated policy programs, the policy concern of a voter becomes relatively more important than the rent concern. Hence, a higher rent level does not reduce as much this party's probability of being in the government with more differentiated policy programs. To sum up, the trade-off when choosing her policy program is between increasing her vote share by choosing a more moderate policy and relaxing the trade-off of the following rent decision stage by choosing a more extreme policy, i.e. by differentiating herself in terms of policy programs.

### 1.1 Related Literature

Polo (1998) considers a two-party competition and finds that parties are not able to extract rents unless there is an uncertainty about voters' preferences. This uncertainty softens the competition, since a party is not anymore sure to win the election even if she is less corrupt than her opponent. Consequently, this uncertainty leads to positive rents in equilibrium. In our paper, we also find positive rents in equilibrium and the basic driving force of our result for parties other than the middle party is the uncertainty in our model, in a very different nature though. In our paper, uncertainty does not originate from voters' preferences, which are perfectly known in our model, but from the inherent uncertainty about the coalition government formation stage, i.e. about which parties will form the coalition government. Clearly, whereas uncertainty about voters' preferences can be always present, uncertainty in coalition formation is often attributed to the proportional rule systems and can be fixed by electoral reform.

Myerson (1993) studies how effective the different electoral rules are to prevent government corruption. This paper analyzes the same problem as ours and calls it corruption. As in our model, voters have perfect information on the corruption levels (which correspond to rent levels in our model) of political parties before the elections. However, as opposed to our model, the corruption levels are exogenous and the policy choice is binary: a party is either affirmative or negative with regards to the policy at hand. He assumes further that there is at least one noncorrupt party on each side of the debate in order to see whether voters
will be able to select only those noncorrupt parties. In this setting, plurality rule is "partly effective", i.e. for each political situation, there is a Nash equilibrium where corrupt parties receive no seats, but there is also a Nash equilibrium where they receive seats due to voters' coordination problems.

In Myerson (1993)'s setting, like in ours, the proportional rule ensures that every party receives seats according to her vote share. He assumes further that the winning majority will be formed by parties with the same policy preferences. Hence, there is no uncertainty on the coalition formation stage and the issue becomes simply which one of the two groups of parties will have the majority of votes. Consequently, no single vote is wasted. Therefore, a voter has no reason to vote for a corrupt party. Hence, proportional rule is "fully effective", i.e. for each political situation, there does not exist any Nash equilibrium where corrupt parties receive seats. However, this result is not robust to the introduction of a more general policy space and of a more general government formation stage. In this case, a voter would not vote for a noncorrupt party if he believes that this party will not be in the government anyway. Hence, we would have coordination problems very similar to those arising under a plurality rule and we would conclude that proportional rule is not "fully effective", but "partly effective". Our paper does not focus on voters' coordination problems. In that sense, we can say that our model studies the case where only three parties are seen by voters as potential candidates for the coalition government. We also endogenize political parties' decisions to be corrupt or not.

Kunicova and Rose-Ackerman (2005) and Persson, Tabellini and Trebbi (2003) confirm empirically our prediction about higher corruption levels in proportional rule systems than in plurality rule systems. However, their explanations are different from ours. For instance, both of them focus on the fact that in plurality rule, individual accountability of politicians is stronger since voters elect directly candidates rather than party lists as in proportional rule systems. In addition, Kunicova and Rose-Ackerman (2005) argue that the bigger number of parties in proportional rule systems may be a disadvantage because it may reduce the incentives of opposing parties of monitoring the government due to free-rider problem. On the contrary, Persson, Tabellini and Trebbi (2003) see the higher number of parties as an advantage due to increased political competition. Kunicova and Rose-Ackerman (2005) claim as well that in the context of coalition politics, an opposing party may not disclose a corrupt behavior of a party in the government, taking into account future opportunities to form a coalition with that party. Finally, Persson, Tabellini and Trebbi (2003) state that in plurality rule systems, seat shares are more sensitive to vote shares than in proportional rule systems and that this may lead to stronger competition and less corruption in plurality rule systems.

Our paper makes a similar argument, however, not on the sensitivity of seat shares to vote shares, but on the sensitivity of government composition to vote shares.

Barro (1973) and Ferejohn (1986) study the effect of future elections on rents of a party already in power, in other words, on an incumbent's rents. Persson, Roland and Tabellini (1997) argue that the control of several incumbents improves via separation of their powers.

To the best of our knowledge, the papers which examined coalition governments took the office-related rent level as an exogenous parameter. These papers include Austen-Smith and Banks (1988), Baron and Diermeier (2001) and Baron (1993) ${ }^{3}$. Moreover, taking also policy programs of parties as an endogenous choice of parties, we are able to analyze the interaction between the policy choice and the rent choice.

Our paper is organized as follows. Section 2 presents the model. Section 3 presents the equilibrium. Section 4 studies the fixed selection rule and contrasts the results with those of the original model. Section 5 concludes. Some proofs are in the appendix.

## 2 The Model

There are three political parties: a left-wing, a middle and a right-wing party, called respectively $L, M$ and $R$. They face a continuum of voters indexed by $x \in[0,1]$.

The game is as follows:
First, each party $j=L, R$ selects $p_{j} \in \mathbf{R}$ such that $p_{L} \leq p_{M}=\frac{1}{2} \leq p_{R}$ where $p_{j}$ is the ideal policy point of party $j$.

Second, each party $j=L, M, R$ selects $r_{j} \in[0, \bar{R}]$ where $r_{j}$ is the amount of political rent party $j$ will take if she is in the government and $\bar{R}$ is the maximum feasible amount of rent a party can extract from the political system.

Third, each voter votes for one of the three political parties.
Fourth, there is a government formation stage as follows: A party $j$ is probabilistically chosen as a formateur, with probabilities given by the parties' vote shares. Then the formateur party $j$ chooses a party $k, k \neq j$ as her coalition partner. The final policy of the coalition $j k$ is $p_{j k}$ where $p_{j k}$ is defined as $\frac{p_{j}+p_{k}}{2}$. Parties $j$ and $k$ respectively get $r_{j}$ and $r_{k}$ as

[^3]rents.
The payoffs of the players (political parties and voters) are as follows:
When a political party $j$ is in the government, her payoff is
\[

$$
\begin{equation*}
v\left(r_{j}, p_{j}, p\right)=r_{j}-\alpha\left(p-p_{j}\right)^{2} \tag{1}
\end{equation*}
$$

\]

where $p$ is the final policy of the government and $\alpha>0$ measures the relative importance of the policy-related payoff.

When a political party $j$ is not in the government, her payoff is 0 .
The utility function of a voter $x$ is

$$
\begin{equation*}
u(x, p, R)=-(p-x)^{2}-R \tag{2}
\end{equation*}
$$

where $x$ is the ideal policy point of the voter $x$, and $R=r_{j}+r_{k}$ is the total rent of the coalition government $j k$. The voters' ideal policy points are uniformly distributed on $[0,1]$.

We call party $i$ 's vote share as $w_{i}$, for $i=L, M, R$.
Now, we have some comments about our model.
First, it is clear that we do not allow single-party governments. In other words, we assume implicitly that no single party will be able to get enough votes to form a singleparty government. This may be due to partisan voters of $L, M$ and $R$ and possibly of other parties not modeled. We see our model as a case where strategic voters see $L, M$ and $R$ as the potential candidates to take part of the coalition government. This approach seems legitimate, given the prevalence of coalition governments and our focus on the effect of coalition governments on political rents.

Second, we assume that the final policy of a coalition is the midpoint of coalition partners' ideal points, since this is the policy which maximizes the joint policy-related utility of the two coalition partners.

The assumption about parties' ideal policies, namely $p_{L} \leq p_{M}=\frac{1}{2} \leq p_{R}$, amounts to fix parties' labels. For instance, party $L$ cannot choose a right-wing policy, i.e. a policy on the right half of the policy space. Setting $p_{M}=\frac{1}{2}$ is not a restriction, since, as it will be seen below, $M$ will be always in government irrespective of her ideal policy, given that she is the middle party. Clearly then, every party would like to be the middle party, but explaining why $M$ succeeds and not the others is out of our scope.

Coming to the payoff structure of a political party, the interpretation is that a party is office-motivated so that she does not get any utility when she is not in the government. The
reason why she cares about the final policy chosen when in government is the re-election concerns as in Austen-Smith and Banks (1988). However, in their paper, the party cares about the final policy even when she is not in government for re-election concerns. This may be criticized on the ground that an out-of-government party cannot be held responsible for the final policy implemented. $\alpha>0^{4}$ measures the degree of these re-election concerns. To put it more clearly, a party can be punished in the next elections by retrospective voters if the final policy is far from her announced policy. Since future elections are not modeled, $\alpha$ is an exogenous parameter.

Our solution concept is subgame perfect Nash equilibrium. Hence, we proceed by backwards induction. To deal with multiple equilibria at the voting stage, we shall present later a simple refinement for that stage.

## 3 Equilibrium

### 3.1 The Government Formation Stage

If party $L$ is chosen as the formateur, she selects party $M$ as the coalition partner rather than party $R$. Indeed, because $p_{L} \leq p_{L M} \leq p_{L R},{ }^{5}$

$$
r_{L}-\alpha\left(p_{L}-p_{L M}\right)^{2} \geq r_{L}-\alpha\left(p_{L}-p_{L R}\right)^{2}
$$

Hence, in this case, the final policy is $p_{L M}$.
Symmetrically, if party $R$ is chosen as the formateur, she selects party $M$ as the coalition partner and the final policy is $p_{R M}$.

If party $M$ is chosen as the formateur, there are three cases. If $p_{L}$ and $p_{R}$ are symmetric around $p_{M}$, she is indifferent between parties $L$ and $R$ as her coalition partner since

$$
r_{M}-\alpha\left(p_{M}-p_{L M}\right)^{2}=r_{M}-\alpha\left(p_{M}-p_{R M}\right)^{2}
$$

In this case, we assume that with probability $\frac{1}{2}, M$ selects $L$ as her coalition partner and the final policy is $p_{L M}$; and with probability $\frac{1}{2}, M$ selects $R$ as her coalition partner and the final policy is $p_{R M}$.

If $p_{L}$ is closer to $p_{M}$ than $p_{R}$ is, $M$ selects $L$ as her coalition partner and the final policy

[^4]chosen is $p=p_{L M}$. Symmetrically, if $p_{R}$ is closer to $p_{M}$ than $p_{L}$ is, $M$ selects $R$ as her coalition partner and the final policy chosen is $p=p_{R M}$.

We note that $M$ is always a coalition member and that the final policy is either $p_{L M}$ or $p_{R M}$.

### 3.2 The Voting Stage

Since there are a continuum of voters, no individual voter is pivotal. Hence, every voting configuration is a Nash equilibrium. To avoid this problem, we propose the following simple refinement.

We assume that every voter considers herself as being of mass $\varepsilon>0$. If a voter has an incentive to deviate for any $\varepsilon>0$, then this is not a voting equilibrium. Actually, every voter is pivotal for any $\varepsilon>0$, since each of them can change the probabilities of the parties being selected as formateur. This feature avoids the genuine problem of the multiplicity of equilibria of elections.

Another possible refinement would be to allow the deviation of arbitrarily small groups of voters, as in Cho (2003). This refinement would lead to the same results.

Lemma 1 Let $K=\frac{r_{R}-r_{L}}{p_{R}-p_{L}}+\frac{p_{L}+p_{R}+1}{4}$. In any voting equilibrium, the resulting coalition government is LM with probability $K$, and $R M$ with probability $(1-K)$.

Proof: When $M$ is indifferent between $L$ and $R$ as her coalition partner, equivalently when $p_{L}$ and $p_{R}$ are symmetric around $p_{M}$, the respective probabilities of the two possible coalition governments $L M$ and $R M$ are $w_{L}+\frac{w_{M}}{2}$ and $w_{R}+\frac{w_{M}}{2}$.

The first term of $L M$ 's probability is the probability of $L$ being chosen as the formateur, which results in the coalition government $L M$ as explained in the previous subsection. The second term is half of the probability of $M$ being chosen as the formateur, since if $M$ is the formateur, the outcome is $L M$ with probability $\frac{1}{2}$.

The same logic applies to $R M$ 's probability.
Suppose we have a voting equilibrium where party $i$ ' vote share is $w_{i}^{*}$. Consider that a voter $x$ of party $L$ deviates and votes for party $M$. Then the vote shares of parties $L$ and $M$ would change to

$$
w_{L}=w_{L}^{*}-\varepsilon
$$

and

$$
w_{M}=w_{M}^{*}+\varepsilon
$$

If the initial situation is an equilibrium, it should be that this deviation is not profitable for voter $x$. Hence,

$$
\begin{gathered}
\left(w_{L}^{*}-\varepsilon+\frac{w_{M}^{*}+\varepsilon}{2}\right)\left(-\left(p_{L M}-x\right)^{2}-r_{L}-r_{M}\right) \\
+\left(w_{R}^{*}+\frac{w_{M}^{*}+\varepsilon}{2}\right)\left(-\left(p_{R M}-x\right)^{2}-r_{R}-r_{M}\right) \leq \\
\left(w_{L}^{*}+\frac{w_{M}^{*}}{2}\right)\left(-\left(p_{L M}-x\right)^{2}-r_{L}-r_{M}\right)+\left(w_{R}^{*}+\frac{w_{M}^{*}}{2}\right)\left(-\left(p_{R M}-x\right)^{2}-r_{R}-r_{M}\right)
\end{gathered}
$$

Equivalently ${ }^{6}$,

$$
\left(-\left(p_{L M}-x\right)^{2}-r_{L}-r_{M}\right) \geq\left(-\left(p_{R M}-x\right)^{2}-r_{R}-r_{M}\right)
$$

This deviation decreases the probability of the coalition government $L M$ and increases the probability of the coalition government $R M$. Hence, we see that this deviation is not profitable for voters who prefer $L M$ rather than $R M$ as the coalition government.

Given that $p_{M}=\frac{1}{2}$ and $p_{j M}=\frac{p_{j}+p_{M}}{2}$ for $j=L, R$, the above inequality is equivalent to ${ }^{7}$

$$
x \leq \frac{r_{R}-r_{L}}{p_{R}-p_{L}}+\frac{p_{L}+p_{R}+1}{4}
$$

The deviation of a voter $x$ from party $L$ to party $R$ is not profitable under the same inequality since the impact of this deviation is the same: decreasing $L M$ 's probability and increasing $R M$ 's probability.

Similarly, the deviation of a voter $x$ of party $R$ for party $L$ or $M$ is not profitable if and only if

$$
x \geq \frac{r_{R}-r_{L}}{p_{R}-p_{L}}+\frac{p_{L}+p_{R}+1}{4}
$$

[^5]The deviation of a voter $x$ of party $M$ for party $L$ is not profitable if and only if

$$
\begin{gathered}
\left(w_{L}^{*}+\varepsilon+\frac{w_{M}^{*}-\varepsilon}{2}\right)\left(-\left(p_{L M}-x\right)^{2}-r_{L}-r_{M}\right) \\
+\left(w_{R}^{*}+\frac{w_{M}^{*}-\varepsilon}{2}\right)\left(-\left(p_{R M}-x\right)^{2}-r_{R}-r_{M}\right) \leq \\
\left(w_{L}^{*}+\frac{w_{M}^{*}}{2}\right)\left(-\left(p_{L M}-x\right)^{2}-r_{L}-r_{M}\right)+\left(w_{R}^{*}+\frac{w_{M}^{*}}{2}\right)\left(-\left(p_{R M}-x\right)^{2}-r_{R}-r_{M}\right)
\end{gathered}
$$

Equivalently,

$$
x \geq \frac{r_{R}-r_{L}}{p_{R}-p_{L}}+\frac{p_{L}+p_{R}+1}{4}
$$

Similarly, the deviation of a voter $x$ of party $M$ for party $R$ is not profitable if and only if

$$
x \leq \frac{r_{R}-r_{L}}{p_{R}-p_{L}}+\frac{p_{L}+p_{R}+1}{4}
$$

Hence, the deviation of a voter $x$ of party $M$ for party $L$ or $R$ is not profitable if and only if

$$
x=\frac{r_{R}-r_{L}}{p_{R}-p_{L}}+\frac{p_{L}+p_{R}+1}{4}
$$

Define $K=\frac{r_{R}-r_{L}}{p_{R}-p_{L}}+\frac{p_{L}+p_{R}+1}{4}$.
Then the voting equilibrium is such that voters with $x \in[0, K]$ vote for $L$, and voters with $x \in(K, 1]$ vote for $R .{ }^{8}$ Hence, the probabilities of coalition governments $L M$ and $R M$ are respectively $K$ and $1-K$.

When $M$ prefers $L$ as her coalition partner, equivalently when $p_{L}$ is closer to $p_{M}$ than $p_{R}$ is, the respective probabilities of the two possible coalition governments $L M$ and $R M$ are $w_{L}+w_{M}$ and $w_{R}$. This is because $L M$ is the coalition government formed when $L$ or $M$ is chosen as formateur, and $R M$ is the coalition government formed only when $R$ is chosen as formateur.

With the same line of reasoning as above, we conclude that voters with $x \in[0, K]$ vote for $L$ or $M$, and voters with $x \in(K, 1]$ vote for $R$. Voters are indifferent between voting for $L$ or $M$, since both choices have the same consequence, namely increasing the probability of the coalition government $L M$. We reach the same threshold value $K$, since voters are concerned with the same problem: Increasing the probability of a coalition government $R M$ or $L M$.

Hence, the probabilities of coalition governments $L M$ and $R M$ are again respectively $K$

[^6]and $1-K$.
Symmetrically, when $M$ prefers $R$ as her coalition partner, equivalently when $p_{R}$ is closer to $p_{M}$ than $p_{L}$ is, we find that voters with $x \in[0, K]$ vote for $L$, and voters with $x \in(K, 1]$ vote for $R$ or $M$. This gives the same probabilities for the two possible coalition governments $L M$ and $R M$. Q.E.D.

The first remark is that when $M$ is indifferent between $L$ and $R$ as her coalition partner, no strategic voter votes for party $M$, since strategic voters expect $M$ to be in government in any case and furthermore voting for $M$ does not change the probabilities of the possible two outcomes.

The second remark is that these expressions do not depend on $r_{M}$. Since voters expect $M$ to be in government in any case, the choice to be made is between parties $L$ and $R$.

The first term of the probability of the coalition government $L M$, i.e. $\frac{r_{R}-r_{L}}{p_{R}-p_{L}}$, implies that when $L$ increases her rent level, she reduces her probability of being in the government. However, this reduction is less important, the more differentiated the parties' policy choices are. The reason is that confronted with more differentiated policy choices, voters are less sensitive to the parties' rent level. The second term, i.e. $\frac{p_{L}+p_{R}+1}{4}$, implies that $L$ has an incentive to choose a moderate policy in order to attract more votes and so to increase her probability of being in the government. Symmetric remarks apply to $R$.

### 3.3 The Rent Decision Stage

The parties maximize their expected utilities with respect to their rent levels.
$M$ sets the maximum possible rent, i.e. $r_{M}=\bar{R}$, since she will be in the coalition government in any case.

The maximization problem of party $L$ is the following

$$
\max _{r_{L} \in[0, \bar{R}]}\left(\frac{r_{R}-r_{L}}{p_{R}-p_{L}}+\frac{p_{L}+p_{R}+1}{4}\right)\left(r_{L}-\alpha\left(p_{L M}-p_{L}\right)^{2}\right)
$$

where the first term in parantheses is the probability of $L$ being in the government and the second term in parantheses is her utility in this case. Since an out-of-government party has a payoff of 0 , there is no other expression in the maximization problem.

Replacing $p_{L M}=\frac{p_{L}+p_{M}}{2}$ and $p_{M}=\frac{1}{2}$, and solving the maximization problem, we have

$$
\begin{equation*}
r_{L}=\frac{r_{R}}{2}+\frac{\alpha\left(1-2 p_{L}\right)^{2}}{32}+\frac{\left(p_{R}-p_{L}\right)\left(p_{L}+p_{R}+1\right)}{8} \tag{3}
\end{equation*}
$$

The basic trade-off is between the probability of being in the government and the extracted rent level when in government. A higher rent level reduces the probability of being in the government, but at the same time, it increases the payoff when in government. The first term shows that the rent of party $L$ increases with the rent of party $R$. When $r_{R}$ is higher, the benefit of a marginal increase in $r_{L}$ is higher since the probability of $L$ being in the government is higher. Hence, rents are strategic complements like prices in a differentiated duopoly. The second term shows that the rent is bigger when the policy choice of party $L$ is farther away from $\frac{1}{2}$. In this case, the policy-related utility when in government is smaller, then a smaller probability of being in government due to a higher rent level becomes less important. The last term implies that given the value of $p_{L}+p_{R}$, the farther away the policy choices of parties $L$ and $R$, the higher the rent of party $L$, since in this case, the two parties become more differentiated and this decreases the reaction of the voters to an increase of the rent level of a party.

A similar approach allows us to solve for the rent level $r_{R}$ chosen by party $R$ :

$$
\begin{equation*}
r_{R}=\frac{r_{L}}{2}+\frac{\alpha\left(1-2 p_{R}\right)^{2}}{32}+\frac{\left(p_{R}-p_{L}\right)\left(3-p_{L}-p_{R}\right)}{8} \tag{4}
\end{equation*}
$$

The remarks about the equation (4) are the reciprocal ones of the equation (3).
From (3) and (4), we get the unique Nash equilibrium ${ }^{9}$ :

$$
\begin{equation*}
r_{L}=\frac{\left(p_{R}-p_{L}\right)\left(p_{L}+p_{R}+5\right)}{12}+\frac{\alpha\left(1-2 p_{L}\right)^{2}}{24}+\frac{\alpha\left(1-2 p_{R}\right)^{2}}{48} \tag{5}
\end{equation*}
$$

and

$$
\begin{equation*}
r_{R}=\frac{\left(p_{R}-p_{L}\right)\left(7-p_{L}-p_{R}\right)}{12}+\frac{\alpha\left(1-2 p_{L}\right)^{2}}{48}+\frac{\alpha\left(1-2 p_{R}\right)^{2}}{24} \tag{6}
\end{equation*}
$$

### 3.4 The Policy Decision Stage

In that stage, parties maximize their expected utilities with respect to their policy choices, foreseeing the impact of their decisions on the subsequent stages of the game analyzed in the above subsections.

The maximization problem of party $L$ is the following

$$
\max _{p_{L} \in \mathbf{R}}\left(\frac{r_{R}-r_{L}}{p_{R}-p_{L}}+\frac{p_{L}+p_{R}+1}{4}\right)\left(r_{L}-\alpha\left(p_{L M}-p_{L}\right)^{2}\right)
$$

After replacing $r_{L}$ and $r_{R}$ by their values in equations (5) and (6), the first-order condition

[^7]of this maximization problem gives the following best reaction function
\[

$$
\begin{equation*}
p_{L}=\frac{p_{R}}{3}+\frac{\alpha-5}{3 \alpha+3} \tag{7}
\end{equation*}
$$

\]

We see that as $R$ chooses a more extreme policy, $L$ chooses a more moderate policy. One of the reasons is that when $R$ chooses a more extreme policy, there is more differentiation resulting in high rent levels, consequently $L$ focuses more on increasing her vote share and therefore her probability of being in government by choosing a more moderate policy. The other reason is that the marginal increase of $L$ 's vote share as a result of a more moderate policy is higher when $R$ 's policy is more extreme.

Moreover, we remark that when $\alpha$ is higher, i.e. when $L$ puts more weight on her policyrelated utility, $L$ chooses a more moderate policy so that she gets a higher policy-related utility when forming a coalition government with $M$. The reason is that when she chooses a more moderate policy, the final policy of the $L M$ coalition government is closer to her ideal policy point.

Similarly, we find the best reaction function of party $R$ :

$$
\begin{equation*}
p_{R}=\frac{p_{L}}{3}+\frac{\alpha+7}{3 \alpha+3} \tag{8}
\end{equation*}
$$

Symmetric remarks apply to $R$.
From equations (7) and (8), we get the unique Nash equilibrium of the policy decision stage ${ }^{10}$ :

$$
p_{L}=\frac{\alpha-2}{2 \alpha+2}
$$

and

$$
p_{R}=\frac{\alpha+4}{2 \alpha+2}
$$

Now, we can replace these values in the equations (5) and (6) to find out the rent levels chosen by the parties in equilibrium. The result is

$$
r_{L}=r_{R}=\frac{33 \alpha+24}{16(1+\alpha)^{2}}
$$

Replacing equilibrium values in (1), the payoff of party $L$ or $R$ when in government is $\frac{3}{2(1+\alpha)}$.

[^8]Hence, we have proved the following proposition.
Proposition 1: Political parties choose divergent policy programs and positive rents.
When parties $L$ and $R$ choose their policy programs, their trade-off is the following: as they move towards a more moderate policy, they have more votes and so more chance to be in the government. However, a more moderate policy leads also to less policy differentiation. Then the voting decision becomes more sensitive to parties' rent levels. Hence, less policy differentiation results in stronger competition at the following rent decision stage. As a result of this trade-off, parties choose divergent policies and consequently, they are able to extract rents. Indeed, if parties chose the same policy, voting decision would become infinitely sensitive to parties' rent levels. Consequently, parties would not be able to extract any rents.

Note that $p_{L}$ is strictly increasing with $\alpha$ and $r_{L}$ is strictly decreasing with $\alpha$. As $\alpha$ increases, parties put more weight on their policy-related utility relative to their rent levels and so they choose a more moderate policy. Consequenly, their rent levels decrease.

As $\alpha$ goes to 0 , a party's only concern becomes extracting rents. In this case, $r_{L}$ and $r_{R}$ go to $\frac{3}{2}, p_{L}$ goes to -1 and $p_{R}$ goes to 2 .

## 4 Fixed Selection Rule

In this variant, we change the government formation stage. All the remaining parts of the model remains the same. The government formation process in this subsection, called fixed selection rule, is as follows:

The party with the highest vote share is selected as formateur. Then this party $j$ chooses a party $k, k \neq j$ as her coalition partner. The final policy of the coalition $j k$ is $p_{j k}$ where $p_{j k}$ is defined as $\frac{p_{j}+p_{k}}{2}$. Parties $j$ and $k$ respectively get $r_{j}$ and $r_{k}$ as rents. If there are more than one party tying for being a formateur, i.e. if they have the same vote share, the more moderate party is selected as formateur ${ }^{11}$. If they are equally moderate, they are selected with equal probabilities.

To avoid complexities which do not affect results, we focus on the equilibrium where $M$ has the smallest vote share.

The main result of this section is that with the fixed selection rule, the equilibrium consists of converging policy platforms and no rent for parties other than the middle party. This result

[^9]is in sharp contrast with the result of the model with the probabilistic selection rule. Given that the fixed selection rule is deterministic, this confirms that this is the probabilistic nature of the coalition government formation process which drives diverging policy platforms and positive rents for parties $L$ and $R$ in equilibrium of the original model.

### 4.1 The Government Formation Stage

After a party is chosen as the formateur, the analysis is the same as in the original model. Hence, calling party $i$ 's vote share as $w_{i}$ and given the fixed selection rule, we have the following lemma:

Lemma 2 If $w_{L}>w_{R}>w_{M}$ or if $w_{R}=w_{L}>w_{M}$ and $L$ is more moderate than $R, L M$ is the coalition government formed and the final policy is $p_{L M}$.

If $w_{R}>w_{L}>w_{M}$ or if $w_{R}=w_{L}>w_{M}$ and $R$ is more moderate than $L, R M$ is the coalition government formed and the final policy is $p_{R M}$.

If $w_{R}=w_{L}>w_{M}$ and, $L$ and $R$ are equally moderate, $L M$ or $R M$ is the coalition government formed with equal probabilities. The final policy is respectively $p_{L M}$ or $p_{R M}$.

### 4.2 The Voting Stage

In case of a probabilistic selection rule, every voter considering himself to be of mass $\varepsilon>0$ is pivotal for any $\varepsilon>0$, since he can change the probabilities of parties' selection as formateur. However, in case of a fixed selection government formation rule, a voter does not consider himself as pivotal if a party's vote share is higher than others' by more than $\varepsilon$, since he cannot change which party will be the formateur. Therefore, in case of a fixed selection rule, there is the usual problem of the multiplicity of voting equilibria. The standard solution of elimination of weakly dominated strategies does not help as usual in multiparty settings.

To solve this problem, we focus on equilibria where the difference of vote shares of parties $L$ and $R$ is at most $\varepsilon>0$ so that a voter can be pivotal. In addition, we consider voting equilibria where $w_{L}, w_{R}>w_{M}+\varepsilon$. As made precise above, this is only to avoid analyzing too many cases which would not change results. We call these equilibria as FSR (fixed selection rule) voting equilibria.

Lemma 3 Define $A$ as $A=\left(p_{L M}-\frac{1}{2}\right)^{2}-\left(p_{R M}-\frac{1}{2}\right)^{2}$.

- If $p_{L} \neq p_{R}$,
- a FSR voting equilibrium such that LM or RM is formed with equal probabilities exists if and only if $r_{R}=r_{L}$ and, $L$ and $R$ are equally moderate.
- a FSR voting equilibrium such that LM is formed with certainty exists if and only if either $r_{L}-r_{R}<-A$ or, $r_{L}-r_{R} \leq-A$ and $A<0{ }^{12}$.
- a FSR voting equilibrium such that RM is formed with certainty exists if and only if either $r_{R}-r_{L}<A$ or, $r_{R}-r_{L} \leq A$ and $A>0$.
- If $p_{L}=p_{R}$,
- a FSR voting equilibrium such that LM or RM is formed with equal probabilities exists if and only if $r_{L}=r_{R}$.
- a FSR voting equilibrium such that LM is formed with certainty exists if and only if $r_{L} \leq r_{R}$.
- a FSR voting equilibrium such that RM is formed with certainty exists if and only if $r_{L} \geq r_{R}$.

Define $x_{I}$ as the ideal policy of the indifferent voter between coalition governments $L M$ and $R M$.

We are looking for an equilibrium where every voter is pivotal in terms of changing the order of parties L's and R's vote shares. Moreover, voters know that the party with the highest vote share will form the government with $M$. Hence, to have an equilibrium where, say, $L$ has the highest vote share, it should be that the majority of voters prefer $L M$ to $R M$.

When $p_{L} \neq p_{R}$, all voters to the left of $x_{I}$ prefer $L M$ to $R M$ and all voters to the right of $x_{I}$ prefer $R M$ to $L M$. Then if $x_{I}>\frac{1}{2}$, equivalently if $r_{R}-r_{L}>A$, the majority of voters prefer $L M$ to $R M$. When $L$ is more moderate than $R, L$ is chosen as formateur if $L$ and $R$ have equal vote shares. Hence, in this case, the inequality becomes weak.

When $p_{L}=p_{R}$, every voter has the same preferences between $L M$ and $R M$, and these preferences depend on parties $L$ 's and $R$ 's rent levels. If, say, $r_{L}<r_{R}$, every voter prefers $L M$ to $R M$, then $L$ has the highest vote share in equilibrium. If $r_{L}=r_{R}$, every voter is indifferent between $L M$ and $R M$, then it is possible to have equilibria where $L$ or $R$ has the highest vote share or where their vote shares are equal. In this case, we choose the equilibrium where $L$ and $R$ have equal vote shares.

[^10]
### 4.3 The Rent Decision Stage

First, we note that $M$ sets the maximum possible rent, i.e. $r_{M}=\bar{R}$ as in the original model, since she will be in the government in any case. Hence, we analyze below the rent levels of parties $L$ and $R$.

We define $c_{L}$ as $c_{L}=\max \left\{0, \alpha\left(p_{L M}-p_{L}\right)^{2}\right\}$, and $c_{R}$ as $c_{R}=\max \left\{0, \alpha\left(p_{R M}-p_{R}\right)^{2}\right\} . c_{L}$ is the minimum rent level which ensures $L$ a non-negative payoff when in government. Given that her payoff is 0 out of government, $L$ never chooses a rent below that minimum level. Remark that $r_{L}=c_{L}>0$ does not mean that $L$ extracts rents, she is just breaking even. The same remarks apply to $R$.

Lemma 4 Relative to voting equilibria with $w_{L}^{*}, w_{R}^{*}>w_{M}^{*}+\varepsilon$ and $\left|w_{L}^{*}-w_{R}^{*}\right|<\varepsilon$,

- if $A=0$, i.e. if $p_{L}$ and $p_{R}$ are symmetric around the median policy, $r_{L}=r_{R}=$ $c_{L}=c_{R}$ is the unique equilibrium of the rent decision stage, and the resulting coalition government is $L M$ or $R M$ with equal probabilities. If $p_{L}=p_{R}=\frac{1}{2}$, then $c_{L}=c_{R}=0$, hence the equilibrium is $r_{L}=r_{R}=0$.
- If $A>0$, i.e. if $p_{R}$ is closer to the median policy than $p_{L}, r_{L}=c_{L}, r_{R}=A+c_{L}$ is the unique equilibrium of the rent decision stage, and the resulting coalition government is RM for sure.
- If $A<0$, i.e. if $p_{L}$ is closer to the median policy than $p_{R}, r_{L}=-A+c_{R}, r_{R}=c_{R}$ is the unique equilibrium of the rent decision stage, and the resulting coalition government is LM for sure.

The main intuition is that if the median voter prefers one coalition to another, it means that the majority of voters prefers this coalition ${ }^{13}$. Then, in the case of the fixed selection rule, the government is formed certainly by this coalition. As a result, in order to ensure being in the government, there is a fierce competition between $L$ and $R$ to be the preferred coalition partner of $M$ in the eyes of the median voter. If $L$ and $R$ have symmetric policy choices around the median policy, competition results in no rents in equilibrium. When a party's policy choice is closer to the median policy, she uses this policy advantage to extract positive rents as well as to guarantee a place in the government.

In the case of probabilistic selection rule, parties $L$ and $R$ can never ensure being in the government, consequently competition is not as harsh as in this case.

[^11]
### 4.4 The Policy Decision Stage

Proposition 2: In the unique equilibrium, $p_{L}=p_{R}=\frac{1}{2}$ and $r_{L}=r_{R}=0$. The resulting coalition government is LM or RM with equal probabilities.

Proof: From the analysis of the rent decision stage, we see that if $p_{R}$ is closer to the median policy than $p_{L}$ is, i.e. if $A>0, R M$ is the resulting coalition government for sure, and vice versa. Hence, parties want their policy choices to be as close as possible to the median policy. Then, the unique equilibrium of the policy decision stage is $p_{L}=p_{R}=\frac{1}{2}$. Then, we know from the previous lemma that $r_{L}=r_{R}=0$, the resulting voting outcome is $w_{L}^{*}=w_{R}^{*}>w_{M}^{*}+\varepsilon$ and the resulting coalition government is $L M$ or $R M$ with equal probabilities. Q.E.D.

There is a harsh competition between $L$ and $R$ to have the highest vote share in order to ensure being in the government. This competition leads to the result that both parties choose the median policy and no rent. No party has an incentive to deviate, since otherwise she is out of the government for sure. There cannot be any other equilibrium, since then a party would be able to ensure being in the government by proposing a more popular policy or a lower rent level.

We conclude once again that it is the probabilistic nature of the coalition formation process which softens political competition and which results in divergent policy choices and positive rents for $L$ and $R$. The deterministic nature of the fixed selection rule results in convergent policy choices and no rent for $L$ and $R$.

## 5 Conclusion

In this paper, we have analyzed the ability of political competition to eliminate political parties' rents in the context of coalition governments. We have shown that the inherent uncertainty about the coalition formation stage results in a relaxed political competition. Consequently, parties are able to extract positive rents. Furthermore, we have shown that if the coalition formation stage is deterministic, political competition is able to eliminate rents except the rent of the middle party which will be a member of all potential coalition governments and hence which cannot be disciplined. Hence, we have confirmed our intuition that the driving reason of positive rents of political parties other than the middle party is the stochastic nature of the coalition formation.

In addition, we have found an original explanation of extreme policy choices of political parties. Since voters are ready to pay more rents with differentiated policy choices of parties, parties differentiate themselves in order to be able to extract more rents. However, parties have also an incentive to choose moderate policies in order to attract more votes. The parties' policy choices reflect this trade-off.

## 6 Appendix

Proof of Lemma 3: First, we are looking for a FSR voting equilibrium such that $L M$ and $R M$ are formed with equal probabilities. This is the case if $w_{L}^{*}=w_{R}^{*}>w_{M}^{*}+\varepsilon$ and, $L$ and $R$ are equally moderate. Consider that a voter $x$ of party $L$ deviates and votes for party $R$ or $M$. Then, the vote shares will be $w_{R}>w_{L}>w_{M}$ and the outcome will be $R M$ for sure. This deviation is not profitable if and only if

$$
\begin{aligned}
& \frac{1}{2}\left[-\left(p_{L M}-x\right)^{2}-r_{L}-r_{M}\right]+\frac{1}{2}\left[-\left(p_{R M}-x\right)^{2}-r_{R}-r_{M}\right] \\
\geq & -\left(p_{R M}-x\right)^{2}-r_{R}-r_{M}
\end{aligned}
$$

equivalently

$$
\begin{equation*}
r_{R}-r_{L} \geq\left(p_{L M}-x\right)^{2}-\left(p_{R M}-x\right)^{2} \tag{9}
\end{equation*}
$$

Similarly, a voter $x$ of party $R$ will not deviate and vote for party $L$ or $M$ if and only if

$$
\begin{equation*}
r_{R}-r_{L} \leq\left(p_{L M}-x\right)^{2}-\left(p_{R M}-x\right)^{2} \tag{10}
\end{equation*}
$$

Similarly, a voter $x$ of party $M$ will not deviate and vote for party $L$ or $R$ if and only if

$$
\begin{equation*}
r_{R}-r_{L}=\left(p_{L M}-x\right)^{2}-\left(p_{R M}-x\right)^{2} \tag{11}
\end{equation*}
$$

If $p_{L} \neq p_{R}$, we see that the right-hand side of inequality (9) is increasing in $x$. Hence, calling the voter $x$ for which $r_{R}-r_{L}=\left(p_{L M}-x\right)^{2}-\left(p_{R M}-x\right)^{2}$ (in other words, the indifferent voter between $L M$ and $R M$ governments) as $x_{I}$, we see that the inequalities (9), (10) and (11) are equivalent respectively to $x \leq x_{I}, x \geq x_{I}$ and $x=x_{I}$.

Hence, if $p_{L} \neq p_{R}$, there is a FSR voting equilibrium such that $L M$ and $R M$ are formed with equal probabilities if and only if $x_{I}=\frac{1}{2}$ and, $L$ and $R$ are equally moderate. Since $A=0$ in this case, this is equivalent to $r_{R}-r_{L}=A=0$. In this equilibrium, voters with $x \in\left[0, \frac{1}{2}\right]$ vote for $L$, and voters with $x \in\left(\frac{1}{2}, 1\right]$ vote for $R$.

If $p_{L}=p_{R}$, then the inequalities (9), (10) and (11) are equivalent respectively to $r_{R}-r_{L}$ $\geq 0, r_{R}-r_{L} \leq 0$ and $r_{R}-r_{L}=0$. Hence, respectively, either every voter prefers $L M$ to $R M$ or every voter prefers $R M$ to $L M$ or every voter is indifferent between $L M$ and $R M$.

Therefore, if $p_{L}=p_{R}$, there is a FSR voting equilibrium ${ }^{14}$ such that $L M$ and $R M$ are formed with equal probabilities if and only if $r_{R}=r_{L}$, in which case any voter $x \in[0,1]$ is indifferent between voting for $L$ or $R$.

Now, we are looking for a FSR voting equilibrium such that $L M$ is the coalition government formed. This is the case either when $w_{L}^{*}>w_{R}^{*}>w_{M}^{*}+\varepsilon$ and $w_{L}^{*}<w_{R}^{*}+\varepsilon$, or when $w_{L}^{*}=w_{R}^{*}>w_{M}^{*}+\varepsilon$ and $L$ is more moderate than $R$. Consider that a voter $x$ of party $L$ deviates and votes for party $R$ or $M$. Then, the vote shares will be $w_{R}>w_{L}>w_{M}$ and the outcome will be $R M$ for sure. This deviation is not profitable if and only if inequality (9) holds.

Now, consider that a voter $x$ of party $M$ deviates and votes for party $R$. Then, the vote shares will be $w_{R}>w_{L}>w_{M}$. Similarly as above, this deviation is not profitable if and only if inequality (9) holds.

There is no other deviation that would change the outcome of the government formation stage.

If $p_{L} \neq p_{R}$, then inequality (9) is equivalent to $x \leq x_{I}$. Hence, a FSR voting equilibrium such that $w_{L}^{*}>w_{R}^{*}>w_{M}^{*}+\varepsilon$ and $w_{L}^{*}<w_{R}^{*}+\varepsilon$ exists if and only if $x_{I}>\frac{1}{2}$, equivalently $r_{R}-r_{L}>A$. A FSR voting equilibrium such that $w_{L}^{*}=w_{R}^{*}>w_{M}^{*}+\varepsilon$ and $L$ is more moderate than $R$ exists if and only if $r_{R}-r_{L} \geq A$ and $A<0$, equivalently $r_{L}-r_{R} \leq-A$ where $-A>0$.

If $p_{L}=p_{R}$, then inequality (9) is equivalent to $r_{R}-r_{L} \geq 0$, in which case every voter prefers $L M$ to $R M$. Therefore, if $p_{L}=p_{R}$, a FSR voting equilibrium ${ }^{15}$ such that $w_{L}^{*}>w_{R}^{*}>w_{M}^{*}+\varepsilon$ and $w_{L}^{*}<w_{R}^{*}+\varepsilon$ exists only if and only if $r_{L} \leq r_{R}$. When $p_{L}=p_{R}, L$ and $R$ are equally moderate, hence a FSR voting equilibrium such that $w_{L}^{*}=w_{R}^{*}>w_{M}^{*}+\varepsilon$ and $L$ is more moderate than $R$ is not possible.

Similarly, it can be shown that if $p_{L} \neq p_{R}$, a FSR voting equilibrium such that $R M$ is the coalition government formed exists if and only if $r_{R}-r_{L}<A$ or, $r_{R}-r_{L} \leq A$ and $A>0$. If $p_{L}=p_{R}$, such an equilibrium exists if and only if $r_{R} \leq r_{L}$. Q.E.D.

Proof of Lemma 4: Define $c_{L}$ as $c_{L}=\max \left\{0, \alpha\left(p_{L M}-p_{L}\right)^{2}\right\}$, and $c_{R}$ as $c_{R}=$

[^12]$\max \left\{0, \alpha\left(p_{R M}-p_{R}\right)^{2}\right\}$.
If $A>0$, then $R$ can always choose $r_{R}=A+r_{L}$ and ensure that she is in the coalition government. Moreover, if $r_{R}>A+r_{L}$, $L$ can choose $r_{L}<r_{R}-A$ and secure being in the government. Hence, $r_{L}=c_{L}, r_{R}=A+c_{L}$ is the unique equilibrium of this case and the resulting coalition government is $R M$ for sure.

If $A<0$, it can be shown similarly that $r_{R}=c_{R}, r_{L}=-A+c_{R}$ is the unique equilibrium and the resulting coalition government is $L M$ for sure.

If $A=0, L$ will never choose $r_{L}$ such that $r_{L}>r_{R}$, since the resulting coalition government is $R M$ for sure. Similarly, $R$ will never choose $r_{R}$ such that $r_{R}>r_{L}$. Hence, in equilibrium, it must be that $r_{L}=r_{R}$. If $r_{L}=r_{R}$, the resulting coalition government is $L M$ or $R M$ with equal probabilities. $r_{L}=r_{R}>c_{L}=c_{R}{ }^{16}$ cannot be an equilibrium, because both $L$ and $R$ have an incentive to decrease a little bit their rent levels in order to be in the coalition government for sure. Then, the only possible equilibrium is $r_{L}=r_{R}=c_{L}=c_{R}$. Indeed, this is an equilibrium, since if $L$ or $R$ deviates and increases her rent level, she will be out of the government for sure. Remark that when $p_{L}=p_{R}=\frac{1}{2}, c_{L}=c_{R}=0$, therefore the equilibrium is $r_{L}=r_{R}=0$. Q.E.D.

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[^1]:    ${ }^{1}$ A well known hypothesis of the literature on coalition governments is that the middle party will be a member of the coalition government. Some empirical support for this hypothesis is presented in Laver and Schofield (1998).

[^2]:    ${ }^{2}$ A famous result of political economy literature, Duverger's Law formalized by Palfrey (1989) and Feddersen (1992) among others, states that a plurality rule system implies a two-party competition.

[^3]:    ${ }^{3}$ Some papers focus on coalition formation, taking election results as given. Baron and Ferejohn (1989) consider a purely distributional issue. Baron (1991) studies a two-dimensional policy issue. Baron (1998) and Diermeier and Feddersen (1998) focus on cohesion strength of a coalition government already in power. Merlo (1997) and Diermeier, Eraslan and Merlo (2003) structurally estimate a stochastic bargaining model of government formation. On the other hand, Besley and Coate (1997) study a citizen-candidate election model, but in their model, only one candidate is chosen and so there is not a legislature in which a (possibly coalition) government is formed.

[^4]:    ${ }^{4}$ When $\alpha=0$, parties are indifferent about their coalition partners. Consequently, many equilibria can be constructed. In particular, the equilibrium when $\alpha>0$ is still an equilibrium when $\alpha=0$.
    ${ }^{5}$ We assume that $L$ chooses $M$ as her partner in case of indifference.

[^5]:    ${ }^{6}$ Remark that this equivalence holds for any $\varepsilon>0$.
    ${ }^{7}$ This is true if $p_{L} \neq p_{R}$, which is satisfied in equilibrium. As we will see, we have to consider the case where $p_{L}=p_{R}$, when we study the fixed selection rule.

[^6]:    ${ }^{8}$ The voter $x=K$ is indifferent between any parties. Here, we assume that she votes for $L$. This assumption is innocuous since any single voter is of mass 0 .

[^7]:    ${ }^{9}$ These values are indeed positive in equilibrium.

[^8]:    ${ }^{10}$ Remark that, depending on $\alpha$, the below equilibrium values of $p_{L}$ and $p_{R}$ may be even outside the support $[0,1]$ of the distribution of voters' ideal policies.

[^9]:    ${ }^{11}$ This assumption is made only to avoid equilibrium problems, similar to those of Bertrand competition models, at the rent decision stage.

[^10]:    ${ }^{12} A<0(A>0)$ is equivalent to say that $L(R)$ is more moderate than $R(L)$.

[^11]:    ${ }^{13}$ Since there is a continuum of voters, if the median voter prefers, say, $L M$ to $R M$, then there exists an $\bar{x}>\frac{1}{2}$ such that all voters with $x<\bar{x}$ prefer $L M$ to $R M$.

[^12]:    ${ }^{14}$ Note that there is a multiplicity of such equilibria. $\left(w_{L}^{*}, w_{R}^{*}, w_{M}^{*}\right)=(0.4,0.4,0.2)$ and $\left(w_{L}^{*}, w_{R}^{*}, w_{M}^{*}\right)=$ $(0.35,0.35,0.3)$ are two examples of such equilibria.
    ${ }^{15}$ Note that there is a multiplicity of such equilibria. $\left(w_{L}^{*}, w_{R}^{*}, w_{M}^{*}\right)=\left(0.4+\frac{\varepsilon}{3}, 0.4-\frac{\varepsilon}{3}, 0.2\right)$ and $\left(w_{L}^{*}, w_{R}^{*}, w_{M}^{*}\right)=\left(0.35+\frac{\varepsilon}{3}, 0.35-\frac{\varepsilon}{3}, 0.3\right)$ are two examples of such equilibria.

[^13]:    ${ }^{16} c_{L}=c_{R}$, since $L$ and $R$ are equally moderate when $A=0$.

