Waveguide Problem : Presentation



The interior problem

$$(\mathcal{P}^{i}) \begin{vmatrix} -\Delta \mathbf{u}^{i} - \boldsymbol{\rho}(\mathbf{x}) \, \omega^{2} \mathbf{u}^{i} = \boldsymbol{f}(\mathbf{x}), & \text{in } \Omega^{i} \\ \nabla \mathbf{u}^{i} \cdot \mathbf{n} = 0 & \text{on } \partial \Omega^{i} \cap \partial \Omega \\ + \frac{\partial \mathbf{u}^{i}}{\partial \mathbf{x}} + T^{+} \, \mathbf{u}^{i} = 0 & \text{on } \Gamma^{+} \\ - \frac{\partial \mathbf{u}^{i}}{\partial \mathbf{x}} + T^{-} \, \mathbf{u}^{i} = 0 & \text{on } \Gamma^{-} \end{vmatrix}$$

The interior problem is of **Fredholm type** (if ρ_{per} is constant near the transverse section).

Conjecture : the DtN operators depends analytically of the frequency (except for a countable set of frequencies)

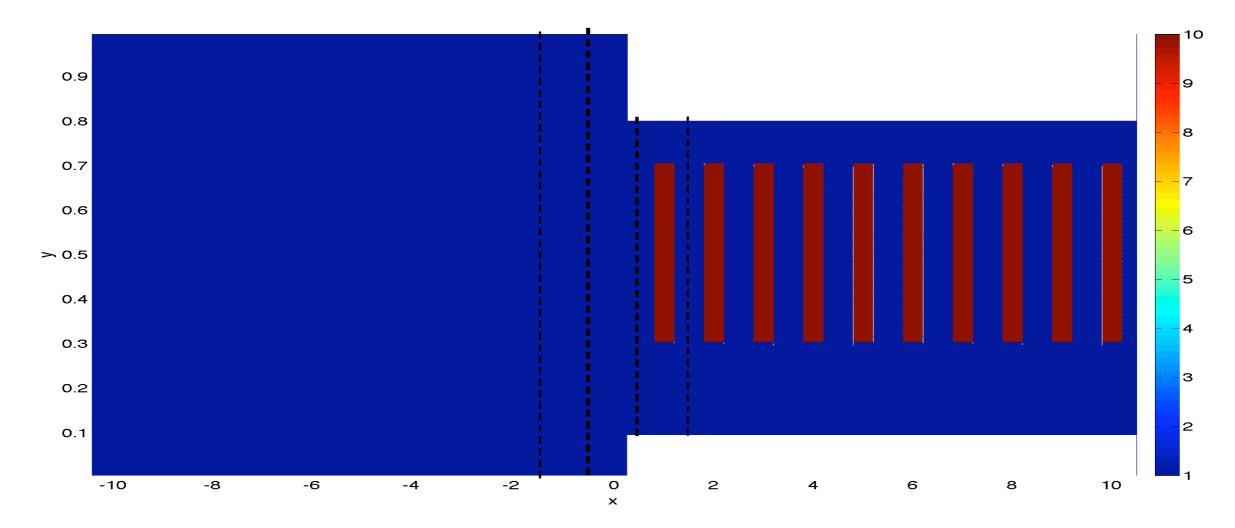
Theorem

The interior problem is **well posed** except for a countable set of frequencies.

Definition of the physical solution of the problem without absorption

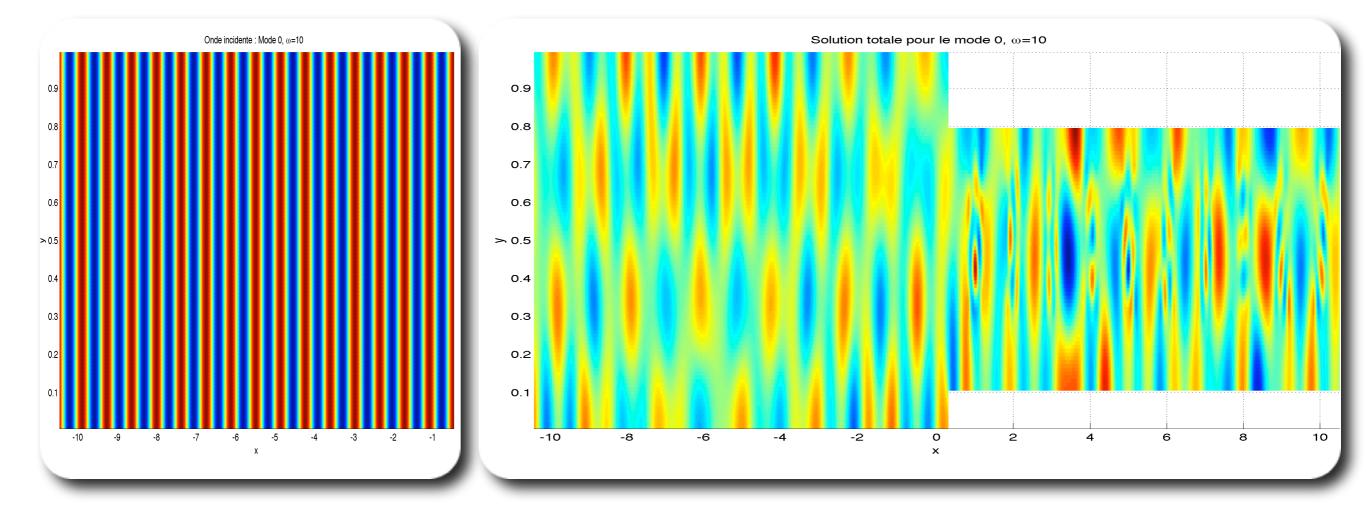
$$\begin{split} & \boldsymbol{u} = \boldsymbol{u}^{i} & \text{in } \Omega^{i} \\ & \boldsymbol{u} = \boldsymbol{u}^{+}(\varphi_{i}^{+}) & \text{in } \Omega^{+} & \text{with } \varphi_{i}^{+} = \boldsymbol{u}^{i}|_{\Gamma^{+}} \\ & \boldsymbol{u} = \boldsymbol{u}^{-}(\varphi_{i}^{-}) & \text{in } \Omega^{-} & \text{with } \varphi_{i}^{-} = \boldsymbol{u}^{i}|_{\Gamma^{-}} \end{split}$$





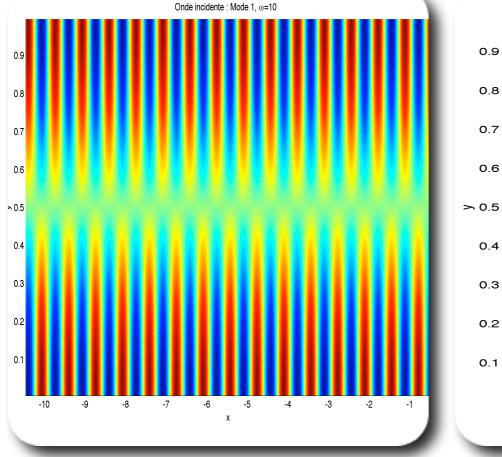
Refraction index

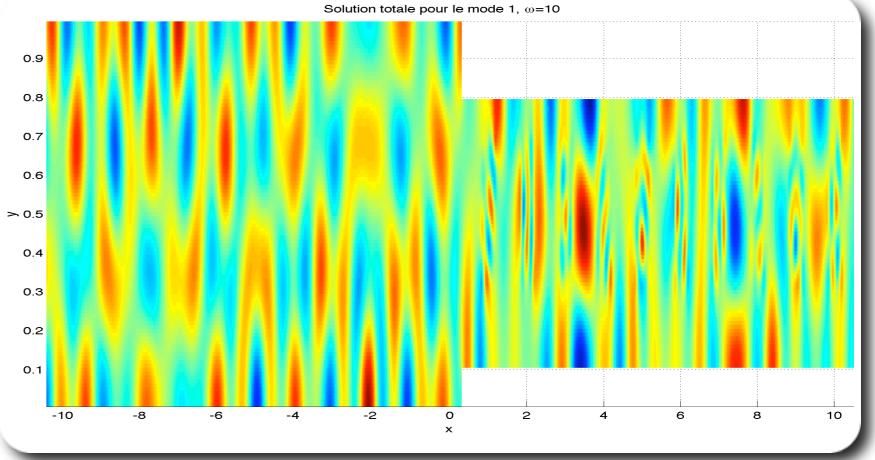




Incident field

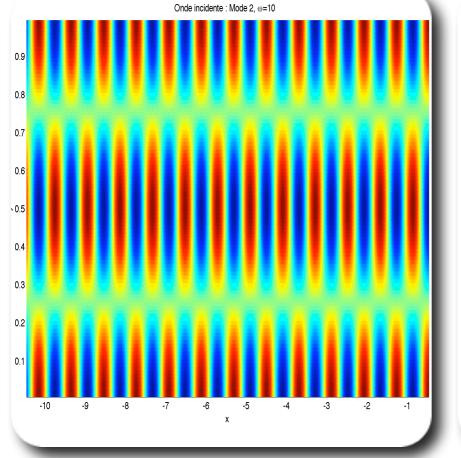


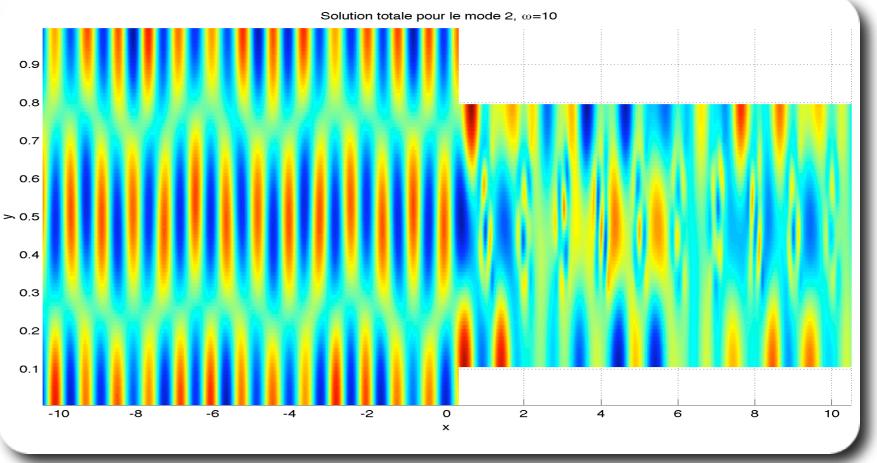




Incident field

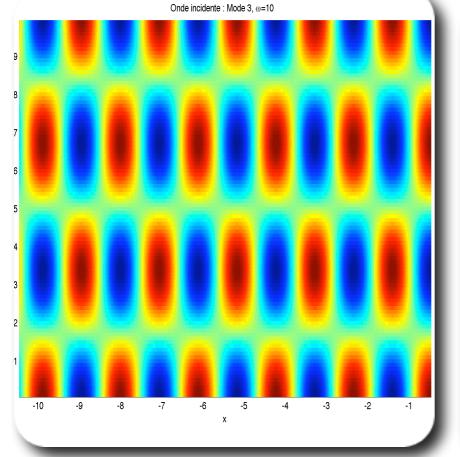


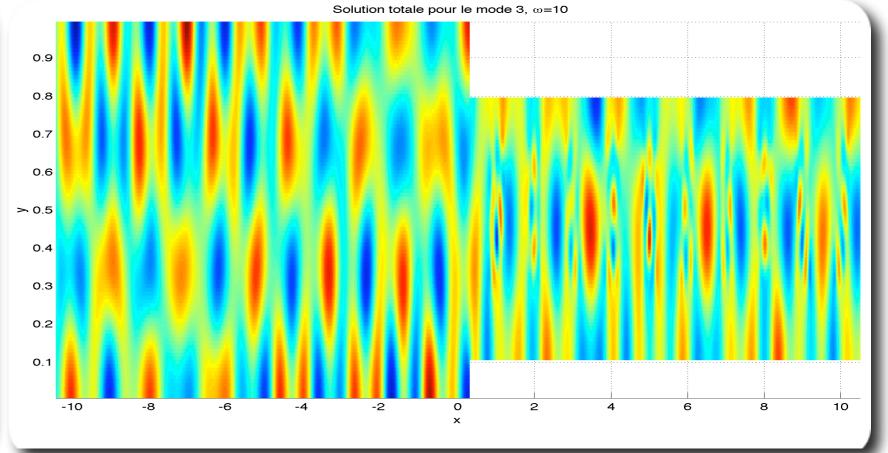




Incident field







Incident field