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| **Office hours** | by appointment |
| **Email** | guhrin@uni-goettingen.de |
| **Class time** | block course, April 3 – April 7; 14:00–18:00 |
| **Location** | MZG 8.163 (14:00–16:00); MZG 5.111 (16:00–18:00) |
| **Webpage** | see StudIP |

## Overview and objectives

The econometrics preparatory course aims to refresh the students' knowledge of matrix algebra and basic statistics necessary to succeed in the Econometrics Module. Assuming a diverse student background, the course will guide through several results in linear (matrix) algebra and statistics with a hands-on, data-oriented approach.

Arguably the most important estimator in econometrics is the ordinary least squares (OLS) estimator for the multiple linear regression model:

$$\hat{\beta} = (X'X)^{-1}X'y = (\sum_{i=1}^{n} x_i x_i')^{-1} \sum_{i=1}^{n} x_i y_i. \tag{1}$$

This formula alone contains several concepts from matrix algebra: *transposition*, *inversion*, *scalar* and *vector multiplication*, *matrix summation*. Furthermore, the *matrix* $X'X$ is known to be *symmetric* and *positive semi-definite*. In order to be able to derive (1) from the basic principles of OLS estimation, one needs to be comfortable with *matrix differentiation*.

As it will be shown in Econometrics I, the OLS estimator $\hat{\beta}$ is 1.) linear, 2.) unbiased, and 3.) efficient (in the class of linear and unbiased estimators). More precisely:

1. $\hat{\beta}$ is a *linear function* of $y$;
2. The *expectation* of the vector $\hat{\beta}$ is the vector $\beta$, i.e., $\mathsf{E}(\hat{\beta}) = \beta$.
3. The *variance-covariance matrix* of the vector $\hat{\beta}$ is *smaller* than that of any other linear, unbiased estimator.

A further result in Econometrics I is that, under certain assumptions, the OLS estimator is also the *maximum likelihood* estimator. Under the same assumptions, we can *test* the significance of $\beta_j$ (an element of the vector $\beta$) with the *t-test*, either by calculating the *p-value*, or calculating a *confidence interval* for the desired *significance level*.

The objective of the course is to fill the concepts typeset in *italics* with life and make students comfortable with manipulating potentially high-dimensional matrices and vectors. The course will consist of non-technical lectures featuring small-scale paper-pencil examples, and separate computer sessions where we apply the concepts through manipulating real-world data matrices ($X$ and $y$) with the R programming language.

## Course outline

1. Vectors and $n$-dimensional spaces: Vectors, manipulations with vectors, orthogonality, linear independence, span, orthogonal complement.
2. Matrix algebra:
   a) Matrices and linear operations with matrices,
   b) Matrix multiplication,
   c) Transpose, trace,
   d) Determinant,
   e) Dimension, rank,
   f) Inverse,
   g) Definiteness, the Löwner ordering,
   h) Differentiation,
   i) Special matrices: square, identity, unit, symmetric, idempotent,
   j) Stochastic matrices: expectation, variance-covariance matrix.
3. Statistics recap:
   a) Testing: on the example of the $t$-test, p-value, confidence interval,
   b) Distributions: Normal, $\chi^2$, $F$,
   c) Maximum likelihood: principle, small illustrative example.

## Readings

The material of the course is standard and can be found in many standard textbooks. Particularly useful, however, are the following resources:

1. Schmidt, Karsten, and Götz Trenkler: *Moderne Matrix-Algebra*, Springer, 1998.
   Chapters: 1.1–1.5; 2.1–2.6, 2.8; 3.1–3.3; 10.1–10.2.

2. Sydsæter, Knut, Peter Hammond, and Arne Strøm: *Essential Mathematics for Economic Analysis*, Pearson, 2012.
   Chapters: 15.2-15.5, 15.7, 15.8; 16.1–16.2, 16.4, 16.6.

3. Harville, David A.: *Matrix Algebra from a Statistician's Perspective*, Springer, 1997.
   Chapters: 1.1–1.3; 3.1–3.2; 4.4; 5.1–5.3; 8.1–8.2; 13.1–13.2; 15.2, 15.4–15.5.

4. Petersen, Kaare B., and Michael S. Pedersen: *The Matrix Cookbook*, Ver. November 15, 2012.
   Freely available online at: https://www.math.uwaterloo.ca/~hwolkowi/matrixcookbook.pdf

5. Casella, George, and Roger L. Berger: *Statistical Inference*, Cengage Learning, 2001.
   Chapters: 3.3; 7.2.2; 8.1, 8.3.1.