Zeroing In On Exclusively Exclusive Content

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Zero = *no*?

Zero and *no* sound pretty synonymous in (1a-b):

(1) a. Zero signals have been detected by our instruments.

b. No signals have been detected by our instruments.

In particular, they have the same (downward) entailments:

(2) a. Zero signals have been detected by our instruments. →
Zero (weak) signals have been detected (this week) by our (most sensitive) instruments.

b. No signals have been detected by our instruments. →
No (weak) signals have been detected (this week) by our (most sensitive) instruments.

Zero ≠ *no*

But unlike *no*, *zero* fails to license NPIs (Zeijlstra, 2007):

(3) a. # Zero signals have been detected by any of our our instruments.
    b. No signals have been detected by any of our our instruments.

(4) a. # Zero signals have ever been detected by our instruments.
    b. No signals have ever been detected by any of our our instruments.

(5) a. # Zero signals at all have been detected by our instruments.
    b. No signals at all have been detected by our instruments.

Zero options?

Bylinina and Nouwen (2018) (BN) show that existing analyses of numerals and plurals either fail to explain the difference between *zero* and *no*, or else fail to return adequate truth conditions for *zero* in the first place, given the standard view that the denotations of plural count nouns have the structure of a join semi-lattice (Link, 1983).

Not an upper-bounding quantificational determiner

Correct truth conditions; incorrect predictions about NPIs:

(6) a. $[\![\mathit{zero}]\!] = \lambda P \lambda Q. \mid P \cap Q \mid = 0$

b. $\mid \{x \mid \textbf{signals}(x)\} \cap \{x \mid \textbf{detected}(x)\} \mid = 0$

Not a cardinality predicate

Contradictory truth conditions:

(7) a. $[\![zero]\!] = \lambda x.\#(x) = 0$

b. $\exists x[\#(x) = 0 \wedge \textbf{signals}(x) \wedge \textbf{detected}(x)]$

Not a number

Same problem, for same reasons:

(8) a. $[\![\textit{zero}]\!] = 0$

b. $[\![\textit{signals}]\!] = \lambda n \lambda x.\#(x) \wedge \textbf{signals}(x)$

c. $\exists x[\#(x) = 0 \wedge \textbf{signals}(x) \wedge \textbf{detected}(x)]$

Not a number-excluding, upper-bounding degree quantifier

Correct truth conditions; incorrect predictions about NPIs:

(9) a. $[\![\textit{zero}]\!] = \lambda P.\forall n > 0 : \neg P(n)$

b. $\forall n > 0 : \neg\exists x[\#(x) = n \wedge \textbf{signals}(x) \wedge \textbf{detected}(x)]$

Not a maximizing, upper-bounding degree quantifier

Undefined, assuming *max* defined only for non-empty sets:

(10) a. $[\![zero]\!] = \lambda P.max\{n \mid P(n)\} = 0$

b. $max\{n \mid \exists x[\#(x) = n \wedge \textbf{signals}(x) \wedge \textbf{detected}(x)]\} = 0$

And if it weren't undefined, it would license NPIs.

A zero solution

BN show that we can provide correct truth conditions for *zero* sentences *and* capture the NPI data if we assume:

- a lower-bounding semantics for numerals
- an analysis of plural count noun denotations as complete lattices, i.e. as containing a "zero" object $\perp$ such that $\#(\perp) = 0$
- obligatory exhaustification
- a "non-triviality" condition on NPI-licensing

BN's analysis: Truth conditions

If $[\![\mathit{signals}]\!]$ includes $\bot$, then (11) is a tautology, because $\bot$ is a part of every $x \in [\![\mathit{signals}]\!]$.

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But it can be rendered contingent in exactly the right way by exhaustification:

(12) $[\![exh]\!] = \lambda p \lambda w . p(w) \wedge \forall p' \in ALT(p) : p \not\models p' \rightarrow \neg p'(w)$

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(13) $\exists x[\#(x) = 0 \wedge \textbf{signals}(x) \wedge \textbf{detected}(x)] \wedge$
$\forall n > 0 : \neg \exists x[\#(x) = n \wedge \textbf{signals}(x) \wedge \textbf{detected}(x)]$

BN's analysis: NPI licensing

BN follow Gajewski 2011 in assuming that weak NPIs require satisfaction of (14-i), where an environment is non-trivially downward-entailing just in case it is downward-entailing and not also upward-entailing:

(14) Given a structure [$_\alpha$ *exh* [$_\beta$ ... [$_\gamma$ NPI] ...]]:
(i) the environment γ is non-trivially downward-entailing in β
(ii) the environment γ is non-trivially downward-entailing in α

BN's analysis: NPI licensing

Assuming that *no* has one of the quantificational denotations considered earlier, (14-i) is satisfied, and NPIs are licensed:

(15) $[\![\textit{no}]\!] =$

a. $\lambda P_{\langle e,t\rangle}\lambda Q_{\langle e,t\rangle}. \mid P \cap Q \mid = \varnothing$

b. $\lambda P_{\langle d,t\rangle}.\forall d > 0 : \neg P(d)$

c. $\lambda P_{\langle d,t\rangle}.\{d \mid P(d)\} = \varnothing$

(16) $[_{\alpha}$ (*exh*) $[_{\beta}$ no$_i$ $[_{\gamma}$ t_i signals (at all) have ever been detected by (any of) our instruments]]]

BN's analysis: NPI licensing

But in the case of *zero* (14-i) is not satisfied, because β is a tautology, and so is both upward- and downward-entailing:

(17) $[_\alpha$ *exh* $[_\beta$ zero$_i$ $[_\gamma$ t_i signals (at all) have ever been detected by (any of) our instruments]]]

(Assume here for parallelism with *no* that number-denoting $[\![$*zero*$]\!] = 0$ is lifted to a quantificational denotation $[\![$*zero*$]\!] = \lambda P_{\langle d,t \rangle}.P(0)$. This doesn't change the truth conditions.)

Summary of BN's analysis

- a lower-bounding semantics for numerals
- an analysis of plural count noun denotations as complete lattices with "zero" element $\perp$ s.t. $\#(\perp) = 0$
- obligatory exhaustification
- non-triviality condition on NPI-licensing

The rest of the talk

Problems for BN's analysis:

- Modification of *zero* by *only*
- NPIs OK when *zero* composes with "abstract" mass nouns

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My alternative proposal:

- upper-bounding, degree quantifier semantics for numerals
- standard analysis of plural count noun denotations
- contingency condition on NPI-licensing
- elimination of prejacent from at-issue content of exh, leaving behind only the exclusive proposition

#*Only zero*

Elliott (2019) observes that modification of *zero* with *only* in sentences like (1a) is bad; the same is true for exclusives like *just* and *solely*:

(18) # Only/just/solely zero signals have been detected by our instruments.

This is a complete mystery on BN's analysis: since the prejacent is a tautology, pretty much any analysis of *only* predicts that *only zero* should be fine, and should mean the same thing as the bare *zero* sentence (assuming exhaustification)

#*Only zero*

(19) $[\![only]\!](p) =$ (see Horn 1996)

a. defined only if p; if defined: $\forall p' \in ALT(p) : p \not\models p' \rightarrow \neg p$

b. $p \wedge \forall p' \in ALT(p) : p \not\models p' \rightarrow \neg p$

c. $\forall p' \in ALT(p) : p \not\models p' \rightarrow \neg p'$

(20) $[\![only]\!]$(**zero-signals-detected**) $=$

a. defined only if $\mathbf{T}$; if defined: $\forall n > 0 : \neg$**n-signals-detected**;

b. $\mathbf{T} \wedge \forall n > 0 : \neg$**n-signals-detected**

c. $\forall n > 0 : \neg$**n-signals-detected**

#*Only zero*

On the other hand, if *zero* sentences are contradictions — which BN showed to be the case on a standard semantics for plurals and an analysis of numerals as numbers or cardinality predicates — failure of modification by *only* follows on either of the analyses in (19a-b):

(21) $[\![\textit{only}]\!]$(**zero-signals-detected**) =

a. # defined only if **F**; if defined $\forall n > 0 : \neg$**n-signals-detected**

b. # **F** $\wedge$ $\forall n > 0 : \neg$**n-signals-detected**

c. $\forall n > 0 : \neg$**n-signals-detected**

Scalar endpoints

Elliott also observes that there is not a blanket ban on modification of *zero* by *only*, and suggests that *only zero* is infelicitous just in the special case that *zero* picks out a scalar endpoint:

(22) a. The water here has only ever risen to zero$_F$ degrees.
b. # The water here has only ever risen by zero$_F$ degrees.

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b. # The water here has only ever risen by zero$_F$ degrees.

My own judgment is that (22b) is OK, and weird only because it is a Manner-violating way of saying that the temperature has never changed. And when we look more closely, we see that *only zero* is fine just when 'zero' picks out a scalar endpoint.

Differential measures in comparatives

(23) a. Guess what the difference in labor cost is to install crappy heat cable versus quality heat cable? Zero. It costs zero dollars more to install long-lasting, efficient ice dam heat tape than it does the cheap stuff.

b. [The Southern Border Wall] is now ZERO inches longer than the Border Wall was that existed before Trump's Inauguration.

c. In June 2024, Donald Trump will be 78 years old, or zero years older than Joe Biden is now.

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(24) a. Infinitely better functionality and if you wait for a sale only 3/4 the price, or worst case scenario only $0 more.

b. For longer lasting polish, you may upgrade to gel polish for only $0 more.

Cost, percent, chance, probability

(25) a. Universidad Teologica del Caribe students pay only $0 to live on campus.

b. Most of the scoring for the Golden Eagles comes from their starting lineup, as only zero percent of Oral Roberts' points come from bench players.

c. ... due to novel drugs invented to combat leukemic cells and rescue the small patients which 10-20 years ago had only zero chance to survive.

d. The model would be fully ranked for any $K \geqslant 3$, except at a few particular parametric values that could occur with only zero probability, due to the noises in the data model.

Abstract vs. concrete mass nouns

(26) a. If the compensation scheme is a fixed salary, the employee puts in only zero effort and the solution is not efficient.

b. I've read the site a lot, but to have just ZERO interest when a woman is basically saying, "Look, I want to hook up. Your place or mine?"

c. Although I am mostly enjoying classics, I honestly have only zero talent in music.

(27) a. The most important characteristic of Tuscan bread is that it contains (#only) ZERO salt.

b. It's safest to drive when you have (#only) zero alcohol in your body

c. When the water reaches full saturation temperature (boiling point), theoretically there is (#only) zero oxygen left in the water.

Abstract vs. concrete mass nouns

Differ in several ways: acceptability with negative quantifiers in Romance (Tovena, 2001), interaction with degree modifiers and comparative morphemes in Wolof (Baglini, 2015), acceptability with modifiers that pick out qualitative gradability in noun denotations (Morzycki, 2012; Francez and Koontz-Garboden, 2017), etc.

Abstract vs. concrete mass nouns

There are different analyses of these nouns, which agree broadly on the idea that they are associated with scales that rank objects in their extensions according to an intensive measure function, in contrast to the extensive measure functions associated with concrete mass nouns and plurals:

(28)
a. $[\![\textit{sympathy}]\!] = \lambda n \lambda x.\textbf{sympathy}(x) \wedge \mu_{int}(x) = n$
b. $[\![\textit{alcohol}]\!] = \lambda n \lambda x.\textbf{alcohol}(x) \wedge \mu_{ext}(x) = n$
c. $[\![\textit{signals}]\!] = \lambda n \lambda x.\textbf{alcohol}(x) \wedge \#(x) = n$

Abstract vs. concrete mass nouns

We can account for the *only* facts if we further assume that abstract mass nouns differ from concrete mass nouns (and plurals) in including objects in their domains that can be mapped by their associated measure function to 0, i.e. that (29a) can be true, but (29a-b) cannot be.

(29) a. $\exists x[\textbf{sympathy}(x) \wedge \mu_{int}(x) = 0]$
b. $\exists x[\textbf{alcohol}(x) \wedge \mu_{ext}(x) = 0]$
c. $\exists x[\textbf{signals}(x) \wedge \#(x) = 0]$

Summary

The pattern of modification of *zero* by *only*:

- plural count nouns and concrete mass nouns behave as predicted by the traditional analysis, i.e. as though their denotations lack objects that are minimal relative to their associated (extensive) measures.
- *chance, probability* etc. and abstract mass nouns behave as expected on BN's analysis, i.e. as though their denotations include objects that are minimal relative to their associated (intensive) measures.

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- *chance, probability* etc. and abstract mass nouns behave as expected on BN's analysis, i.e. as though their denotations include objects that are minimal relative to their associated (intensive) measures.

Conclusion: The traditional analysis of plural count noun (and concrete mass noun) denotations as join semi-lattices is correct!

Aside

Why can *zero* compose with mass nouns in the first place?

Because unlike other numerals, its meaning doesn't presuppose measurement in terms of units of some sort (cardinalities, kilograms, heads, etc.), which (for mass nouns) needs to be provided by a classifier or similar expression (cf. Krifka, 1989).

Review of BN's analysis

(30) a. # Zero signals have ever been detected.

b. $\exists x[\#(x) = 0 \wedge \textbf{signals}(x) \wedge \textbf{detected}(x)]$

1. $[\![signals]\!]$ includes a zero object $\perp$ in its domain.
2. $[\![zero]\!]$ returns the lower-bounded truth conditions in (30b)
3. since $\perp$ is a part of every $x \in [\![signals]\!]$, (30b) is a tautology
4. composition with *exh* derives contingent (and DE) truth conditions, but this is not enough to license NPIs

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The *only* facts show that $\perp \notin [\![$*signals*$]\!]$, but the logic of BN's account is correct: given 2, *zero N* should fail to license NPIs for any *N* such that $\perp \in [\![N]\!]$.

Abstract mass nouns

- The *only* facts indicated that abstract mass nouns contain objects in their domain that are minimal relative to μ_{int}.
- Assume with Francez and Koontz-Garboden (2017) that the mereology of these nouns tracks μ_{int}, and so $\bot \in [\![sympathy]\!]$.
- If $[\![zero]\!]$ introduces lower-bounded truth conditions, then (31a) should be a tautology and NPIs should be bad:

(31) a. There is zero doubt about the facts.

b. $\exists x[\textbf{doubt}(x) \wedge \mu_{int}(x) = 0]$

Zero doubt at all

(32)
a. I have zero sympathy for any voter of the red wall that voted for this lying charlatan and suffers because of his shite policies.
b. I have zero patience for any person who believes that they are "making a difference" by generalizing and spreading hate about a specific group of people.
c. I had zero familiarity with any characters in the main cast.
d. I have zero respect for any parent fighting against masks in schools.

Zero doubt at all

(33) a. The bones, which seem to have zero possibility of ever knowing life again, reveal how Israel feels about their discipline and exile.

b. Many widows and widowers have zero interest in ever engaging in a romantic relationship with another person.

c. I'm thinking of your highness
And crying long upon the loss
I've found
And on the plus and minus
It's a zero chance of ever
Turning this around. (Soundgarden 'Zero Chance', from the *Down on the Upside* album)

Zero doubt at all

(34)
a. Today I'll show you how to make your Kraft Mac and Cheese Better and DELICIOUS with almost ZERO effort at all!
b. I suppose it can sound a bit shocking to hear someone say with absolute certainty — with zero doubt at all — that there is nothing scary in the world and limitations don't exist.
c. I'm the biggest Rangers fan so trust me when I say they have zero chance at all as long as Quinn is still there coach.

Upshot

(35) a. # There is zero doubt at all.
b. $\exists x[\textbf{doubt}(x) \wedge \mu_{int}(x) = 0]$

1. $[\![\textit{doubt}]\!]$ includes a zero object $\perp$ in its domain.
2. $[\![\textit{zero}]\!]$ returns the lower-bounded truth conditions in (35b)
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The challenge

Assumption:

- $\bot \in [\![\textit{sympathy}]\!]$
- $\bot \notin [\![\textit{messages}]\!]$

(36) a. I have no/zero sympathy for anyone.

b. $[_\alpha$ (*exh*) $[_\beta$ no/zero$_i$ $[_\gamma$ I have t_i sympathy for anyone]]]

c. $[\![\textit{no/zero}]\!](\lambda n.\exists x[\ \ldots\ \textbf{sympathy}(x) \wedge \mu_{int}(x) = n\ \ldots\ ])$

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Aside: This is a good denotation for *no*, however! (Alrenga and Kennedy, 2014)

Analytical option #2: *max*

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But (41) can be lowered to a number-denoting denotation...

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The basic denotation works for abstract mass nouns:

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And the lowered denotation is defined for plural count nouns:

(46) a. $\iota(BE([\![zero]\!]))(\lambda n.\exists x[\ \ldots\ \mathbf{messages}(x) \wedge \#(x) = n\ \ldots\ ])$
b. $\exists x[\ \ldots\ \mathbf{messages}(x) \wedge \mu_{int}(x) = 0\ \ldots\ ]$

Analytical option #2: *max* → 0

(44) a. $[\![zero]\!] = \lambda P.max\{n \mid P(n)\} = 0$
b. $\iota(BE([\![zero]\!])) = 0$

The basic denotation works for abstract mass nouns:

(45) a. $[\![zero]\!](\lambda n.\exists x[\ \ldots\ \textbf{sympathy}(x) \wedge \mu_{int}(x) = n\ \ldots\ ])$
b. $max\{n \mid \exists x[\ \ldots\ \textbf{sympathy}(x) \wedge \mu_{int}(x) = n\ \ldots\ ]\} = 0$

And the lowered denotation is defined for plural count nouns:

(46) a. $\iota(BE([\![zero]\!]))(\lambda n.\exists x[\ \ldots\ \textbf{messages}(x) \wedge \#(x) = n\ \ldots\ ])$
b. $\exists x[\ \ldots\ \textbf{messages}(x) \wedge \mu_{int}(x) = 0\ \ldots\ ]$

But derives contradictiory truth conditions!

Analytical option #3: *sup*

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For a non-empty, finite S, $\sup(S) = 0$ only for $\{0\}$, so we get the right truth conditions for abstract mass nouns:

(48) a. $[\![zero]\!](\lambda n.\exists x[\ \ldots\ \mathbf{sympathy}(x) \wedge \mu_{int}(x) = n\ \ldots\ ])$

b. $sup\{n \mid \exists x[\ \ldots\ \mathbf{sympathy}(x) \wedge \mu_{int}(x) = n\ \ldots\ ]\} = 0$

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And sup$(\varnothing) = -\infty$, so (47) is defined for plural count nouns:

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But it also derives contradictory truth conditions!

Analytical option #4: degree functions

Another option would be to treat abstract mass nouns as degree functions of some sort, without individual arguments at all. If we do this the right way, we could potentially say that $[\![zero]\!] = 0$.

Since I don't want to get into the weeds of noun denotations, I won't pursue this here.

And in any case, for plural count nouns, we will still end up with contradictory truth conditions!

Taking stock

- There are a few different ways to derive correct, NPI-licensing truth conditions for *zero* when it composes with an abstract mass noun.

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Taking stock

- There are a few different ways to derive correct, NPI-licensing truth conditions for *zero* when it composes with an abstract mass noun.
- Each of these ways also predicts that *zero* should give rise to contradictory truth conditions when it composes with a plural count noun (or concrete mass noun).
- **My claim**: This is why *zero* does not license NPIs with these nouns! The account is basically the same as BN's, with one slight modification.

Contingent downward-entailment

(50) Given a structure $[_\alpha$ *exh* $[_\beta$ ... $[_\gamma$ NPI] ...]]:

(i) the environment γ is non-trivially downward-entailing in β

(ii) the environment γ is non-trivially downward-entailing in α

Contingent downward-entailment

(50) Given a structure $[_{\alpha}$ *exh* $[_{\beta}$ ... $[_{\gamma}$ NPI] ...]]:

(i) the environment γ is contingently downward-entailing in β

(ii) the environment γ is contingently downward-entailing in α

Smiley (1959); Clark (1967):

(51) p contingently entails q, $p \vDash q$, iff $p \rightarrow q$ is a tautology, and either:

(i) neither $\neg p$ nor q are tautologies, or

(ii) $p \rightarrow q$ is a substitution instance of some tautology $p' \rightarrow q'$, where neither $\neg p'$ nor q' are tautologies.

Zero licensing explained

(50-i) is satisfied in (52) but not in (53):

(52) a. I have zero sympathy for anyone.
b. $[_\alpha$ (*exh*) $[_\beta$ zero$_i$ [I have t_i sympathy for anyone]]]
c. $max\{n \mid \exists x[\ \dots\ \textbf{sympathy}(x) \wedge \#(x) = n\ \dots\ ]\} = 0$

(53) a. # I have zero messages for anyone.
b. $[_\alpha$ (*exh*) $[_\beta$ zero$_i$ [I have t_i messages for anyone]]]
c. $\exists x[\ \dots\ \textbf{messages}(x) \wedge \#(x) = 0\ \dots\ ])$

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c. $\exists x[\ \ldots\ \textbf{messages}(x) \wedge \#(x) = 0\ \ldots\ ])$

Upshot: Same basic account of NPI licensing as BN. This leaves us with one remaining issue to resolve....

From contradiction to contingency

(12) $[\![exh]\!] = \lambda p \lambda w . p(w) \wedge \forall p' \in ALT(p) : p \not\models p' \rightarrow \neg p'(w)$

Recall that BN derive contingent truth conditions for *zero* sentences through exhaustification. This works when the prejacent is a tautology:

(54) $\exists x[\#(x) = 0 \wedge \textbf{signals}_{\bot}(x) \wedge \textbf{detected}(x)] \wedge$
$\forall n > 0 : \neg \exists x[\#(x) = n \wedge \textbf{signals}_{\bot}(x) \wedge \textbf{detected}(x)]$

From contradiction to contingency

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Recall that BN derive contingent truth conditions for *zero* sentences through exhaustification. This doesn't work when the prejacent is a contradiction:

(54) $\exists x[\#(x) = 0 \wedge \textbf{signals}_{\not\leq}(x) \wedge \textbf{detected}(x)] \wedge$
$\forall n > 0 : \neg\exists x[\#(x) = n \wedge \textbf{signals}_{\not\leq}(x) \wedge \textbf{detected}(x)]$

The problem is that exhaustification entails the prejacent.

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(54) $\exists x[\#(x) = 0 \wedge \textbf{signals}_{\not\perp}(x) \wedge \textbf{detected}(x)] \wedge$
$\forall n > 0 : \neg \exists x[\#(x) = n \wedge \textbf{signals}_{\not\perp}(x) \wedge \textbf{detected}(x)]$

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(**NB:** assuming contigent entailment!)

Exclusively exclusive content

"If the force of the exclusive proposition is to exclude everything other than what is named in or by the subject-term from 'sharing in the predicate,' that is no reason for reading in an implication that something named by the subject-term does 'share in the predicate;' and we certainly cannot exclude from our logic predicables that are not true of anything." — Geach 1962, p. 208

"pex"

Bassi et al. (2021) argue for an analysis of *exh* in which the exclusive component is not part of the operator's at-issue content, but is rather a presupposition:

(55) $\lambda p \lambda w : \forall p' \in ALT(p) : p \not\models p' \rightarrow \neg p'(w).p(w)$

This solves a number of problems for the traditional analysis of *exh*, such as why upper bounding inferendes disappear in downward entailing contexts.

Exhaustification *is* exclusion

My proposal: The semantic content of *exh* is just the exclusive proposition. This content can be *either* not-at-issue or at-issue:

(56) $[\![exh]\!] =$

a. $\lambda p \lambda w : \forall p' \in ALT(p) : p \not\models p' \rightarrow \neg p'(w).p(w)$ **pex**

b. $\lambda p \lambda w . \forall p' \in ALT(p) : p \not\models p' \rightarrow \neg p'(w)$ **excl**

pex is the default; entailment of the prejacent just reflects the absence of any contribution of *exh* to at-issue content.

excl is the special case, and returns exclusively exclusive at-issue content. It is this version of *exh* that is active in *zero* sentences.

Restricting **excl**

Evidently **excl** has a rather limited distribution:

- **excl**$(p \vee q)$ would allow for both p and q to be false.
- If Chierchia (2013) is right about NPIs, **excl** would render (57) contingent, and true just in case Kim detected no signals.

(57) # Kim detected any signals

a. **excl**$(\exists x \in D_{max}[\mathbf{detected}(x)(\mathbf{k}) \wedge \mathbf{signals}(x)])$

b. $\forall D' \subseteq D_{max} : \neg\exists x \in D'[\mathbf{detected}(x)(\mathbf{k}) \wedge \mathbf{signals}(x)]$

Fact

In the preceding cases the prejacent is contingent; in *zero* sentences it is a contradiction.

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- Given that **pex** returns the content of the prejacent its exclusive at-issue contribution, it is natural to assume that such a use *also* presupposes the contingency of the prejacent.
- At the same time, it equally natural to assume that **excl** cannot possibly come with such a contingency presupposition.

Maximize Presupposition to the rescue

If this is correct, then Maximize Presupposition (Heim, 1991; Sauerland, 2008) constrains the interpretation of *exh*:

- *exh* = **pex** whenever the prejacent is contingent
- *exh* = **excl** only in the special case of "predicables that are not true of anything."*

*Or, of course, when they are true of everything!

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- Plural count nouns (and concrete mass nouns) do not include "zero objects" in their domains, but
- Abstract mass nouns do (or the equivalent)
- If abstract mass nouns are otherwise semantically similar to other nouns, then *zero* introduces upper-bounds
- Composition of *zero* with a plural count noun (or concrete mass noun) derives contradictory truth conditions
- Contingent truth conditions come from exhaustification, the semantic content of which is just the exlusive proposition:

(56) $[\![exh]\!] =$

a. $\lambda p \lambda w : \forall p' \in ALT(p) : p \not\models p' \rightarrow \neg p'(w).p(w)$ **pex**

b. $\lambda p \lambda w . \forall p' \in ALT(p) : p \not\models p' \rightarrow \neg p'(w)$ **excl**

Viruses and hosts

Morzycki (2017) suggests that *zero* is a “semantic virus” — a kind of add-on to the basic vocabulary and meanings of English, with unusual properties that sets it outside the core vocabulary and meanings of numerals, plurals, measure terms and so forth.

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- Are my conclusions too big for such a small, weird word?

As we all know all too well, viruses thrive based of compatibility with their hosts, and take advantage of existing structures and operations.

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