Negation and the zero of magnitude

This paper presents evidence in support of the idea that there is a natural language expression of what Russell (1903) calls the *zero of magnitude*, or *quantitative zero*, distinct from sentential negation, the null set and the number zero. The zero of magnitude ($zero_m$) expresses the cardinality of the concept it combines with, namely zero; I suggest that it is realized in English as determiner *no* on some of its occurrences. Informally speaking, the logical form of a sentence such as *Mary has no cats* according to this analysis is 'Mary has $zero_m$ cats', rather than 'it is not the case that Mary has a cat', as it would be on the view that the meaning of negative indefinites such as *no* is translated by sentential negation and an existential quantifier (or a free variable bound by an existential quantifier) (cf. e.g. Ladusaw 1992, McNally 1998, Penka 2010 and Zeijlstra 2004).

The assumption that *no* may be translated by $zero_m$, rather than sentential negation and an indefinite sheds light on contrasts in the use of determiner *no* and the negative particle *not* and on the meaning of nominal expressions that combine with them. For example, the *there*-sentence in English appears to be dedicated to the expression of cardinality. It is possible to view the well-known restrictions on the postverbal nominal in this construction as a requirement for a cardinal expression (rather than a strictly quantificational expression), such that even when no such determiner is present, cardinality is interpreted. For example:

1. There are dogs at that park. =There are *sm* dogs at that park.
2. There is red on the wall. =There is *sm* red on the wall.
3. There is coffee in the cupboard. =There is *sm* coffee in the cupboard.

The assumption that a cardinality expression is required in this environment explains the restriction against sentential negation with bare nouns. 4–6 are infelicitous except as denials of a previous utterance, just as they are when the positive polarity item *some* is realized.

4. There are not (some) dogs in the park. Denial only
5. There is not (some) red on the wall.
6. There is not (some) coffee in the cupboard.

Analyses that decompose *no* into a sentence negation and an indefinite cannot straightforwardly distinguish the different uses of sentences containing *no* from occurrences of sentence negation and an indefinite that are not the result of lexical decomposition. For example, whereas a proposition may be denied by means of sentence negation *not*, *no* is used to deny a quantity, not the proposition.

7. A: Three men are standing on the corner.
   B: Three men are not standing on the corner. Denial of proposition

B': No men are standing on the corner. Denial of quantity

On the other hand, to deny a presupposition of existence we use *no* instead of *not*, singling out the expression that fails to refer as a concept whose magnitude is zero, rather than denying the entire proposition. A denial of the entire proposition is infelicitous in this context.

8. A: The king of France is bald.
   B: No he isn't. There is no king of France.
   B': No he isn't. #There is not a king of France.

The meaning of nominal expressions differs with *no* and *not*. For example, there is a difference in the meaning of a nominal expression as a name of a general term. Consider 9 and 10, with modals: with *no* the extension of the nominal expression is not necessarily empty in all worlds, and the conjunction in 9 is non-contradictory. The conjunction in 10, with *not*, requires contrast to avoid a contradictory reading.

9. There can be no king of France and there can be a king of France.
10. There cannot be a king of France and there can be a king of France.

What is more, although *not* can interact with scalar meaning when applied to *three people* (13, 14), implying "not even three," *no* does not have this effect.

11. No three people could be more kindly received.
12. ??Not three people could be more kindly received.
13. There were not three people at the restaurant.
14. There weren't three people in the restaurant.
15. ?There were no three people at the restaurant.

In fact, Alrenga and Kennedy (2014) also argue that a quantitative *no*, what they call a *negative degree quantifier*, captures the interpretations of English comparatives with *no more* better than a free sentential negation, providing independent support for the present account.

References

Alrenga, P. and C. Kennedy. (2014). *No more* shall we part: Quantifiers in English comparatives. *Natural Language Semantics* 22: 1–53.

Ladusaw, W. (1992). Thetic and Categorical, Stage and Individual, Weak and Strong. Paper presented at SALT IV.

McNally, L. (1998). Existential Sentences without Existential Quantification. *Linguistics and Philosophy* 21: 353–392.

Penka, D. (2010). *Negative Indefinites*. Oxford: Oxford University Press.

Russell, B. (1903). *Principles of Mathematics*. Cambridge: Cambridge University Press.

Zeijlstra, H. (2004). *Sentential negation and Negative Concord.* Ph.D. thesis, University of Amsterdam.