

Recursion and complexity

(3rd lecture)

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Implicit recursion-theoretic approach

Functions have two sorts of input positions, normal and safe:
 $f(\bar{x}; \bar{y})$.

Input-sorted initial functions \mathcal{SI} :

- ▶ ϵ , $S_0(; x)$ and $S_1(; x)$ (the constructores of the algebra);
- ▶ $P(; x)$ (predecessor);
- ▶ $C(; x, y, z_0, z_1)$ (case distinction);
- ▶ $\pi_j^{m;n}$ (projections over both input sorts).

Input-sorted composition sc: $f(\bar{x}; \bar{y}) = g(\bar{r}(\bar{x};) ; \bar{s}(\bar{x}; \bar{y}))$.

[\mathcal{SI} ; SC, **Input-sorted Recursion**]

Implicit recursion-theoretic approach: FPtime

$$\text{FPtime} = [\text{SI}; \text{SC}, \text{SR}] \quad (\text{Bellantoni-Cook 1992})$$

SR (Input-sorted recursion over \mathbb{W}):

$$f(\epsilon, \bar{x}; \bar{y}) = g(\epsilon, \bar{x}; \bar{y})$$

$$f(z_0, \bar{x}; \bar{y}) = h(z_0, \bar{x}; \bar{y}, f(z, \bar{x}; \bar{y}))$$

$$f(\textcolor{blue}{z1}, \bar{x}; \bar{y}) = h(\textcolor{blue}{z1}, \bar{x}; \bar{y}, f(z, \bar{x}; \bar{y}))$$

Implicit recursion-theoretic approach: **FPTIME**

FPtime = [SI; SC, SR] (Bellantoni-Cook 1992)

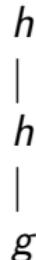
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Example: $f(11)$ leads to $h(11, h(1, g(\epsilon)))$



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h SR reproduces the sequential
| structure of deterministic
h computations.
|
g

Implicit recursion-theoretic approach

Input-sorted composition SC: $f(\bar{x}; \bar{y}) = g(\bar{r}(\bar{x};) ; \bar{s}(\bar{x}; \bar{y}))$.

If $F(x; y)$ is in the class then

- ▶ $f(x, y;) = F(x; y)$ is in the class;
- ▶ $f(; x, y) = F(x; y)$ is NOT in the class.



Implicit recursion-theoretic approach: **FPtime**

SR:

$$f(\epsilon, \bar{x}; \bar{y}) = g(\epsilon, \bar{x}; \bar{y})$$



$$f(z0, \bar{x}; \bar{y}) = h(z0, \bar{x}; \bar{y}, f(z, \bar{x}; \bar{y}))$$

$$f(z1, \bar{x}; \bar{y}) = h(z1, \bar{x}; \bar{y}, f(z, \bar{x}; \bar{y}))$$

► Concatenation: $\oplus(\epsilon; x) = x$

$$\oplus(y0; x) = S_0(; \oplus(y; x))$$

$$\oplus(y1; x) = S_1(; \oplus(y; x))$$

$\oplus(y, x;)$ is in the class.

$\oplus(; y, x)$ is NOT in the class.

Implicit recursion-theoretic approach: **FPtime**

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$\oplus(; y, x)$ is NOT in the class.

► \exp such that $|\exp(z)| = 2^{|z|}$

$$\exp(\epsilon) = 1$$

$$\exp(zi) = \oplus(\exp(z), \exp(z))$$

Implicit recursion-theoretic approach: **FPtime**

SR:

$$f(\epsilon, \bar{x}; \bar{y}) = g(\epsilon, \bar{x}; \bar{y})$$



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