Do Lead-Lag Effects Affect Derivative Pricing?

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Current Version: March 2005

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Abstract

In this paper we address the implementation of pricing models for derivatives whose prices depend on the covariance between asset returns. We show that lead-lag effects can have a strong impact on correlation-dependent derivatives. Simple adjustments to the annualized variance-covariance matrix of finite holding-period returns are derived that account for predictability. As an illustration, we apply our results to the valuation of index-linked stock option plans.

Key Words: Derivative Pricing, Lead-Lag Effects, Predictability

JEL Classification: G13

At first glance, the predictability of an asset's return has no impact on prices of derivatives written on that asset. In the continuous-time, no-arbitrage pricing framework pioneered by Black and Scholes (1973) and Merton (1973), the drift rate of the asset price process under the physical measure does not enter pricing formulae. Predictability[1], however, is usually expressed through a particular drift specification.

Lo and Wang (1995) argue forcefully that predictability and drift specification do matter. For a fixed unconditional variance of finite holding-period asset returns, they show that changes in predictability alter the value of the infinitesimal or instantaneous variance parameter σ^2. As σ^2 enters option pricing formulae, predictability has an impact on the option price. Lo and Wang demonstrate that this impact can be strong for realistic return autocorrelations. Their findings have important consequences for the implementation of option pricing models. Predictability must be taken into account when volatility is estimated from a time series of discretely

[1] There are various possible reasons for predictability. In any case, predictability can be perfectly consistent with market efficiency (see e.g. LeRoy (1989)). This issue was also discussed in the context of co-integration (see e.g. Dwyer and Wallace (1992)).

sampled asset prices. In particular, the annualized sample variance of finite holding-period returns is not a consistent estimator of the instantaneous variance parameter. Consistency requires an autocorrelation adjustment, which Lo and Wang derive for their reference model, the trending Ornstein-Uhlenbeck (O-U) process.

The main contribution of our paper is an investigation into the impact of predictability on derivatives whose prices depend on the covariance between asset returns. There is a growing market for such correlation products (see Mahoney (1995)) and we think that the issue of predictability is even more important for these derivatives than for options written on a single asset. There are two main reasons for this importance. First, implied covariance parameters are rarely available due to the lack of other covariance dependent derivatives. Therefore, we have to rely on time series estimates, where the issue of predictability steps in. Second, we show that there is a close link between the instantaneous covariance and the cross-autocorrelation of finite holding-period returns. Because lead-lag effects, i.e. non-zero cross-autocorrelations, appear frequently[2], predictability affects many covariance dependent derivatives.

The starting point of our analysis in Section 1 is the question of how to obtain the relevant input parameters for derivative pricing. As far as time series estimation is concerned, we have to know how the desired instantaneous parameters are related to the properties of finite holding-period returns. To answer this question, we need a model of the asset price dynamics which is flexible enough to explain important characteristics of asset returns, in particular the commonly observed lead-lag patterns. Section 2 presents such a model. To capture realistic lead-lag relations, we generalize the bivariate trending O-U process proposed by Lo and Wang (1995) in a way that allows for general feedback. We also introduce an alternative parametrization of the model, which helps us to address the issues of stationarity, non-

[2]Lead-lag patterns of financial asset returns have been documented in many studies. Cross-autocorrelations between different stocks or stock portfolios are reported by e.g. Lo and MacKinlay (1990), Conrad, Kaul and Nimalendran (1991) and Badrinath, Kale and Noe (1995). Studies which find lead-lag patterns for international stock markets include Eun and Shim (1989), Arshanapalli and Doukas (1993), Richards (1995), Soydemir (2000), and Masih and Masih (2001). Lead-lag relations between stock indices and stock index futures are investigated by e.g. Kawaller, Koch and Koch (1987), Stoll and Whaley (1990), and Chan (1992). Kwan (1996) documents that stock returns lead yield changes of bonds issued by the same firm. Baillie and Bollerslev (1989) find a co-integration vector in a set of exchange rates, which also implies some form of lead-lag relation.

stationarity, and co-integration. Based on the model presented in Section 2, we derive the relation between the instantaneous variance-covariance parameters and the variance-covariance matrix of finite holding-period returns in Section 3. The understanding of this relation is useful for two reasons. First, we can investigate if and how instantaneous parameters and derivatives' prices are affected by different lead-lag patterns for otherwise fixed moments of finite holding-period returns. In particular, such an analysis helps us to identify and understand situations when price effects are very large. Second, we can analyze how the bias of the annualized sample variance-covariance matrix as an estimator of the instantaneous parameters alters with changing lead-lag patterns. The latter point raises the question of whether adjustments to the annualized sample variance-covariance matrix exist that use only autocorrelations and cross-autocorrelations and are therefore easy to implement. In Section 4, we derive such adjustments for three cases. Section 5 deals with an application of our analysis, the valuation of index-linked stock option plans. We provide numerical examples and a case study that stress the importance of lead-lag effects for derivative pricing. In Section 6 we summarize the main results and give our conclusions.

1 Derivative Pricing and Variance-Covariance Parameters

If we assume competitive, frictionless, and arbitrage-free markets, in which securities trade in continuous time, we can easily derive a valuation equation for the price F of a derivative instrument whose pay-off depends solely on asset prices and time t. Let P and S be the prices of two traded assets whose log-price processes satisfy the following bivariate stochastic differential equation:

$$\begin{aligned} d\log(P(t)) &\equiv dp(t) = \mu_p(\cdot)\ dt + \sigma_p\ dW_p \\ d\log(S(t)) &\equiv ds(t) = \mu_s(\cdot)\ dt + \sigma_s\ dW_s. \end{aligned} \tag{1}$$

The drift coefficients $\mu_p(\cdot)$ and $\mu_s(\cdot)$ can be functions of P, S, t, or the prices of other traded assets.[3] The diffusion coefficients $\sigma_p \geq 0$ and $\sigma_s \geq 0$ are constants, and W_p

[3]Restrictions are imposed only by the regularity conditions which ensure the existence of a solution to the stochastic differential equation (1).

and W_s are two standard Wiener processes such that $dW_p \; dW_s = \rho_{ps} dt$. Let us further assume a known, constant, risk-free interest rate r.

Under our assumptions, we know from the Black-Scholes analysis that $F = F(P, S, t)$ is the solution to the partial differential equation

$$\frac{\partial F}{\partial t} + rP\frac{\partial F}{\partial P} + rS\frac{\partial F}{\partial S} + \frac{1}{2}\sigma_p^2 \; P^2\frac{\partial^2 F}{\partial P^2} + \frac{1}{2}\sigma_s^2 S^2\frac{\partial^2 F}{\partial S^2} + \sigma_p\sigma_s\rho_{ps}PS\frac{\partial^2 F}{\partial P\partial S} = rF, \quad (2)$$

subject to some appropriate terminal and boundary conditions. Of the coefficients in equation (1), only the variances σ_p^2, σ_s^2, and the covariance $\sigma_p\sigma_s\rho_{ps}$ seem to affect derivatives' prices, since the drift coefficients do not appear in the valuation equation. If we know σ_p^2, σ_s^2, and $\sigma_p\sigma_s\rho_{ps}$, we can solve for F, depending on the pay-off structure of the derivative instrument, either analytically or numerically. However, an important question is how to obtain the parameter values.

In principle, there are two estimation approaches. The first one is to imply the variance-covariance parameters from other derivative prices. This approach is not feasible if there are no liquid markets for other derivatives whose theoretical prices also depend on σ_p^2, σ_s^2, and $\sigma_p\sigma_s\rho_{ps}$. In particular, implied estimates of the covariance will often not be available.

In the following analysis, we concentrate on the second approach, which is to estimate the parameters from time series data of asset prices. Assume that we observe prices P and S at n equally spaced intervals of length τ over the period $[0, T]$. Hence, we observe $P(l\tau)$, $S(l\tau)$, with $l = 0, \ldots, n$ and $T = n\tau$. Let us further define continuously compounded returns as $\Delta p_k \equiv \log\big(P(k\tau)/P((k-1)\tau)\big)$ and $\Delta s_k \equiv \log\big(S(k\tau)/S((k-1)\tau)\big)$, $k = 1, \ldots, n$. A natural, easily calculable estimator of σ_p^2, σ_s^2, and $\sigma_p\sigma_s\rho_{ps}$ is the sample variance-covariance matrix of returns, divided by τ.

$$\begin{aligned}
\hat{\sigma}_p^2 &\equiv \frac{1}{n\tau}\sum_{k=1}^{n}(\Delta p_k - \frac{1}{n}\sum_{j=1}^{n}\Delta p_j)^2 \\
\hat{\sigma}_s^2 &\equiv \frac{1}{n\tau}\sum_{k=1}^{n}(\Delta s_k - \frac{1}{n}\sum_{j=1}^{n}\Delta s_j)^2 \qquad (3)\\
\sigma_s\widehat{\sigma_p}\rho_{ps} &\equiv \frac{1}{n\tau}\sum_{k=1}^{n}(\Delta p_k - \frac{1}{n}\sum_{j=1}^{n}\Delta p_j)(\Delta s_k - \frac{1}{n}\sum_{j=1}^{n}\Delta s_j).
\end{aligned}$$

The estimator defined by equations (3) is the maximum likelihood estimator if the vectors $(\Delta p_k, \Delta s_k)$, $k = 1, \ldots, n$, are IID normal random variables. The latter condition holds by construction when $\tau \to 0$. It also holds for an arbitrary interval length τ if the drift coefficients $\mu_p(\cdot)$ and $\mu_s(\cdot)$ in equation (1) are constant over time, i.e. the log-price process follows a bivariate Brownian motion with drift.

However, the bivariate Brownian motion model is not compatible with both lead-lag effects and autocorrelated returns. If we observe lead-lag patterns between finite holding-period returns, it is unclear how the estimates obtained from equations (3) relate to the desired parameters. In order to analyze this relation and to derive simple modifications to estimator (3), we have to specify a more flexible reference model of the price dynamics.

2 A Two-Factor Linear Diffusion Model

A reference model of price dynamics should be sophisticated enough to generate realistic lead-lag patterns, while being simple enough to be analytically tractable. The following variant of model (1), with explicit drift specification, is a natural candidate:

$$\begin{aligned} dp(t) &= (-\alpha_p p(t) - \alpha_s s(t))\ dt + \sigma_p\ dW_p \\ ds(t) &= (-\beta_p p(t) - \beta_s s(t))\ dt + \sigma_s\ dW_s. \end{aligned} \tag{4}$$

where α_p, α_s, β_p, and β_s are constant parameters.[4]

Model (4) is a bivariate extension of the univariate trending O-U log-price process analyzed by Lo and Wang (1995). It encompasses familiar models as special cases. For example, with $\alpha_p = \alpha_s = \beta_p = \beta_s = 0$, we obtain a (correlated) bivariate Brownian motion with drift. Two (correlated) trended O-U processes result if $\alpha_s = \beta_p = 0$, $\alpha_p > 0$, and $\beta_s > 0$. In general, we allow for feedback between $p(t)$ and $s(t)$ through the drift specification, i.e. both α_s and β_p can be non-zero.[5] As the special cases show, model (4) generates both difference-stationary and trend-stationary processes, depending on the parameter values.

[4]Equations (4) should be understood as the detrended versions of the more general trending bivariate O-U log-price process. For our purposes it suffices to look at the detrended process, as a deterministic time trend has no impact on return variances, covariances, autocorrelations, and cross-autocorrelations.

[5]Model (4) is more general than the bivariate trending O-U process of Lo and Wang (1995), in which only one of the parameters can be non-zero.

The stochastic differential equations of model (4) are not well defined for all values of the parameters α_p, α_s, β_p, and β_s. For example, when $\alpha_s = \beta_p = \beta_s = 0$, we require $\alpha_p \geq 0$, since the $p(t)$ process is otherwise explosive.

In order to characterize the range of sensible parameter values, we express the processes $p(t)$ and $s(t)$ as linear combinations of two simpler processes, $x(t)$ and $y(t)$:

$$\begin{aligned} p(t) &\equiv x(t) + a \cdot y(t) \\ s(t) &\equiv b \cdot x(t) + y(t), \end{aligned} \tag{5}$$

where a and b are scalars with $a \cdot b \neq 1$. The latter condition excludes the trivial case that $p(t)$ and $s(t)$ are identical up to a scale factor. $x(t)$ and $y(t)$ satisfy the following stochastic differential equations:

$$\begin{aligned} dx(t) &= -\gamma_x x(t)\ dt + \sigma_x\ dW_x \\ dy(t) &= -\gamma_y y(t)\ dt + \sigma_y\ dW_y, \end{aligned} \tag{6}$$

with $\gamma_x \geq 0$, $\gamma_y \geq 0$, $\sigma_x \geq 0$, $\sigma_y \geq 0$, and $dW_x dW_y = \rho_{xy} dt$. Thus, the $x(t)$ and $y(t)$ processes are either O-U processes for positive γ-parameters or Brownian motions for zero γ-parameters.

From representation (5) of $p(t)$ and $s(t)$, we can directly exclude explosive processes by demanding $\gamma_x \geq 0$ and $\gamma_y \geq 0$. When $\gamma_x > 0$ and $\gamma_y > 0$, both $p(t)$ and $s(t)$ are stationary. When $\gamma_x = \gamma_y = 0$, both $p(t)$ and $s(t)$ are non-stationary. For $\gamma_x = 0$, $\gamma_y > 0$, and $b \neq 0$, the bivariate system is co-integrated, because both $p(t)$ and $s(t)$ are non-stationary due to the Brownian motion component, but $bp(t) - s(t)$ is stationary.[6] Therefore, representation (5) allows us to address how stationarity, non-stationarity, and co-integration of asset prices impacts the lead-lag structure of returns, variance-covariance parameter estimation, and derivatives' prices.

As we show in Appendix 1, imposing the restrictions $\gamma_x \geq 0$ and $\gamma_y \geq 0$ on the system of equations (6) translates into the following restrictions on α_p, α_s, β_p, and β_p:

$$(\alpha_p - \beta_s)^2 + 4\alpha_s\beta_p \geq 0 \tag{7}$$

$$(\alpha_p + \beta_s) - \sqrt{(\alpha_p - \beta_s)^2 + 4\alpha_s\beta_p} \geq 0. \tag{8}$$

[6]Duan and Pliska (2004) note that the formulation of co-integrated diffusion systems is not obvious. However, in our common-factor representation (5) the notion of co-integration becomes straightforward.

Equations (7) and (8) show that the product $\alpha_s \, \beta_p$ must not be too large in absolute terms. It follows from equation (8) that a necessary condition for non-explosive processes is $\alpha_p + \beta_s \geq 0$. When either α_s or β_p is zero, both α_p and β_s must be non-negative.

In the analysis that follows we can either work with the parameter set $\{\alpha_p, \alpha_s, \beta_p, \beta_s, \sigma_p, \sigma_s, \rho_{ps}\}$ or the parameter set $\{a, b, \gamma_x, \gamma_y, \sigma_x, \sigma_y, \rho_{xy}\}$.[7] The advantage of the former set is the more intuitive parameter interpretation. However, the latter parametrization leads to more parsimonious expressions for the moments of the processes $p(t)$ and $s(t)$.

As model (4) is the reference point for our study of the relation between lead-lag effects, variance-covariance estimation and derivative prices, some insight into the kind of lead-lag patterns it generates might be helpful. Figure 1 shows four examples of the unconditional first order return cross-autocorrelations as functions of the holding period τ, measured in years. All examples use the common parameter values $\sigma_p^2 = \sigma_s^2 = 0.3$ and $\rho_{ps} = 1/3$. The drift parameters α_p, α_s, β_p, and β_s take either the values 10, -10, or 0. In the first example, $\alpha_p = 10$, $\alpha_s = 0$, $\beta_p = 0$, and $\beta_s = 10$. This choice of parameters implies that $p(t)$ and $s(t)$ are (correlated) O-U processes. Since both α_s and β_p equal zero, there is no feedback in the drift. The second example uses $\alpha_p = 0$, $\alpha_s = -10$, $\beta_p = 0$, and $\beta_s = 10$. This parameter set describes a situation where $s(t)$ is an O-U process but $p(t)$ is a non-stationary process. The third example, $\alpha_p = 10$, $\alpha_s = -10$, $\beta_p = 0$, and $\beta_s = 0$, can be interpreted as a situation with partial feedback: $s(t)$ is a Brownian motion, but $p(t)$ adapts to $s(t)$ because of the error-correction term $-10(p(t) - s(t))$. The two processes are co-integrated. In the last example, with $\alpha_p = 10$, $\alpha_s = -10$, $\beta_p = -10$, and $\beta_s = 10$, the processes $s(t)$ and $p(t)$ are also co-integrated, but the adaptation is symmetric. We refer to this case as "full symmetric feedback".

[Figure 1 about here.]

The upper half of the figure depicts $\rho^{\Delta}_{p_{-1}s} \equiv Cor(\Delta p_{-1}, \Delta s)$, and the lower half $\rho^{\Delta}_{ps_{-1}} \equiv Cor(\Delta p, \Delta s_{-1})$. Note that from now on we will skip the index k when we deal with unconditional moments, as the unconditional return distribution is identical for all $k \in (1, \ldots, n)$.

[7] The formal relation between the two parameter sets is elaborated in Appendix 1.

In the "no feedback" case, both cross-autocorrelations are negative due to the positive instantaneous correlation and the negative autocorrelations of the O-U processes. When τ goes to infinity, values of $-1/6$ are reached, which is the limit of the autocorrelations multiplied by ρ_{ps}. In the second example, with one O-U process and one non-stationary process, $p(t)$ has a stochastic drift rate of $10s(t)$. This feature explains the positive correlation between Δs_{-1} and Δp. In the third case of co-integration and partial feedback, $s(t)$ is a Brownian motion and $\rho^{\Delta}_{p_{-1}s} = 0$. The positive lead of the $s(t)$ process is due to the error-correction term $-10(p(t) - s(t))$. Finally, for the case with full symmetric feedback we see that cross-autocorrelations are positive in general. The examples show that very different cross-autocorrelation patterns can result from model (4), and that all combinations of signs are possible.

3 Drift Specification and Variance-Covariance Parameters

We are now prepared to analyze the relation between the instantaneous variance-covariance parameters σ_p^2, σ_s^2, $\sigma_p \sigma_s \rho_{ps}$, the estimates obtained from equations (3), and the variance-covariance matrix of finite holding-period returns. If more and more data points become available[8], estimator (3) converges to the unconditional variances $Var[\Delta p]$, $Var[\Delta s]$ and the covariance $Cov[\Delta p, \Delta s]$, divided by τ. Thus, if we establish the link between the unconditional moments of finite holding-period returns and the instantaneous variance-covariance parameters under the price dynamics (4), we cannot only study the impact of different lead-lag patterns on the instantaneous variance-covariance parameters, but also on the asymptotic behavior of estimator (3).

Equations (5) lead to

$$\begin{pmatrix} Var[\Delta p] \\ Var[\Delta s] \\ Cov[\Delta p, \Delta s] \end{pmatrix} = \begin{pmatrix} 1 & a^2 & 2a \\ b^2 & 1 & 2b \\ b & a & (1+ab) \end{pmatrix} \begin{pmatrix} Var[\Delta x] \\ Var[\Delta y] \\ Cov[\Delta x, \Delta y] \end{pmatrix}, \tag{9}$$

where $Var[\Delta x]$ and $Var[\Delta y]$ are the unconditional variances of changes of x and y, respectively, over an interval of length τ, and $Cov[\Delta x, \Delta y]$ is the corresponding

[8]Formally, we lengthen the data period $[0, T]$, but fix the length of the sampling interval τ. Hence, the number of observations n increases.

covariance. In the following analysis, $A = A(a, b)$ denotes the 3×3 matrix on the right-hand side of equation (9). Under the O-U processes (6), the unconditional variances and covariance amount to[9]:

$$
\begin{array}{rclcl}
Var[\Delta x] & = & \frac{1-e^{-\gamma_x \tau}}{\gamma_x \tau}\sigma_x^2\tau & \equiv & g^x(\gamma_x, \tau)\sigma_x^2\tau, \\
Var[\Delta y] & = & \frac{1-e^{-\gamma_y \tau}}{\gamma_y \tau}\sigma_y^2\tau & \equiv & g^y(\gamma_y, \tau)\sigma_y^2\tau, \\
Cov[\Delta x, \Delta y] & = & \frac{2-e^{-\gamma_x \tau}-e^{-\gamma_y \tau}}{(\gamma_x+\gamma_y)\tau}\sigma_x\sigma_y\rho_{xy}\tau & \equiv & g^{xy}(\gamma_x, \gamma_y, \tau)\sigma_x\sigma_y\rho_{xy}\tau.
\end{array}
\tag{10}
$$

By substituting the expressions on the right hand side of equations (10) for $Var[\Delta x]$, $Var[\Delta y]$, and $Cov[\Delta x, \Delta y]$ in equation (9), by using the relation $(\sigma_x^2, \sigma_y^2, \rho_{xy}\sigma_x\sigma_y)^T = A^{-1}(\sigma_p^2, \sigma_s^2, \rho_{ps}\sigma_p\sigma_s)^T$, and by dividing both sides by τ we obtain:

$$
\begin{pmatrix} \frac{Var[\Delta p]}{\tau} \\ \frac{Var[\Delta s]}{\tau} \\ \frac{Cov[\Delta p, \Delta s]}{\tau} \end{pmatrix} = A \begin{pmatrix} g^x & 0 & 0 \\ 0 & g^y & 0 \\ 0 & 0 & g^{xy} \end{pmatrix} A^{-1} \begin{pmatrix} \sigma_p^2 \\ \sigma_s^2 \\ \rho_{ps}\sigma_p\sigma_s \end{pmatrix}. \tag{11}
$$

Equation (11) provides the required relation between the annualized variance-covariance matrix of finite holding-period returns and the instantaneous variance-covariance parameters. For fixed variances and covariance of finite holding-period returns, equation (11) shows how different lead-lag patterns change the instantaneous variances and covariance through g^x, g^y, g^{xy} and A. Therefore, we can study how derivatives' prices are affected by predictability.

Moreover, equation (11) shows how the asymptotic properties of estimator (3) are affected by predictability. As we see from equation (11), the components of estimator (3) are, in general, linear combinations of all three instantaneous parameters. Thus, there will be a bias, even asymptotically. This bias disappears only when $g^x = g^y = g^{xy} = 1$, i.e. when the holding period τ tends to zero, or when $\gamma_x = \gamma_y = 0$ and model (4) is reduced to a bivariate Brownian motion with drift. We note that A, g^x, g^y, and g^{xy} depend on a, b, γ_x, γ_y, and τ only, and are independent of the instantaneous variance-covariance parameters σ_p, σ_s, and ρ_{ps} themselves.

As an example of the size of potential effects on instantaneous parameter values and on the asymptotic bias of estimator (3), we compare the Brownian motion model with the four drift specifications used in Figure 1. Table 1 shows $\sigma_p^2, \sigma_s^2, \sigma_p\sigma_s\rho_{ps}, \rho_{ps}$, and the price of an option to exchange one asset (S) for another (P). τ equals

[9] A derivation can be obtained from the authors upon request.

1/12, i.e. we use monthly returns. The annualized unconditional return variances are 0.3, and the covariance is 0.1. Option prices are calculated according to the pricing formula (12), derived by Margrabe (1978), using the respective instantaneous variance-covariance parameters, asset prices of $P = S = 40$, and a time to expiration $T - t$ of one year.

$$\mathit{Option\ Price} = P \cdot N(d_1) - S \cdot N(d_2), \tag{12}$$

with

$$d_1 \equiv \frac{\ln(P/S) + (\sigma_p^2 + \sigma_s^2 - 2\sigma_p\sigma_s\rho_{ps})(T-t)/2}{\sqrt{(\sigma_p^2 + \sigma_s^2 - 2\sigma_p\sigma_s\rho_{ps})(T-t)}},$$

$$d_2 \equiv d_1 - \sqrt{(\sigma_p^2 + \sigma_s^2 - 2\sigma_p\sigma_s\rho_{ps})(T-t)},$$

and $N(\cdot)$ as the cumulative distribution function of the standard normal distribution. The parameter values and the option price for the reference case appear in the first column of Table 1, which corresponds to the Brownian motion model. In this case we have no lead-lag effects and no asymptotic bias of estimator (3).

In the "no feddback" case with two O-U processes, as shown in the second column, the correlation is unchanged in comparison to the reference case, but the two instantaneous variances and the covariance are higher by the same percentage value. This result holds in general when $\gamma_x = \gamma_y > 0$; then the functions g^x, g^y, and g^{xy} are all equal to $g \equiv \frac{1-e^{-\gamma_x \tau}}{\gamma_x \tau} < 1$ and the instantaneous parameter values are $\frac{1}{g}$ multiplied by the annualized finite holding-period second moments. The parameters σ_p^2, σ_s^2, and ρ_{ps} are all affected in the case of one O-U process and one non-stationary process, as shown in the third column. Here, σ_p^2 is much lower and both σ_s^2 and ρ_{ps} are higher than in the reference case. The results for the case of partial feedback in the fourth column show that we might obtain the same instantaneous variances, but much lower covariances than for the Brownian motion model. The last case of full symmetric feedback points in the same direction, as the correlation parameter is most severely affected. Correlation drops far below 1/3 and even becomes negative. The last two cases, where the logs of the asset prices are co-integrated, warn us to be careful with correlation dependent derivatives. As the last row of Table 1 shows, there are price differences of up to 40% for a simple exchange option compared with the case with no lead-lag effects.

[Table 1 about here.]

4 Some Simple Correction Formulae

The examples in Table 1 show that the drift specification of the asset price processes can have large effects on option prices and the asymptotic bias of the finite holding-period annualized variance-covariance matrix as an estimator of the instantaneous parameters. In the analysis to follow, we derive some simple adjustments for autocorrelations and lead-lag patterns to this estimator. These adjustments provide easily accessible alternatives to the more demanding estimation of the full model (4).[10] Moreover, as the adjustments depend on return cross-correlations and autocorrelations only, we can gain some insight into the effect of predictability on a particular derivative pricing problem by simply looking at sample moments.

We first determine the first order autocovariances $Cov(\Delta p, \Delta p_{-1})$ and $Cov(\Delta s, \Delta s_{-1})$, as well as the cross-autocovariances $Cov(\Delta p, \Delta s_{-1})$ and $Cov(\Delta s, \Delta p_{-1})$.

From model (5) it follows

$$\begin{pmatrix} Cov[\Delta p, \Delta p_{-1}] \\ Cov[\Delta s, \Delta s_{-1}] \\ Cov[\Delta p, \Delta s_{-1}] \\ Cov[\Delta s, \Delta p_{-1}] \end{pmatrix} = \begin{pmatrix} 1 & a^2 & a & a \\ b^2 & 1 & b & b \\ b & a & 1 & ab \\ b & a & ab & 1 \end{pmatrix} \begin{pmatrix} Cov[\Delta x, \Delta x_{-1}] \\ Cov[\Delta y, \Delta y_{-1}] \\ Cov[\Delta x, \Delta y_{-1}] \\ Cov[\Delta y, \Delta x_{-1}] \end{pmatrix}, \tag{13}$$

where $Cov[\Delta x, \Delta x_{-1}]$, $Cov[\Delta y, \Delta y_{-1}]$, $Cov[\Delta x, \Delta y_{-1}]$, $Cov[\Delta y, \Delta x_{-1}]$ are the first order autocovariances and cross-autocovariances of Δx and Δy.

Furthermore, under the O-U processes x and y in equations (6), we can derive the autocovariances and cross-autocovariances on the right-hand side of equation (13):

$$\begin{array}{rcl} Cov[\Delta x, \Delta x_{-1}] & = & -\frac{[1-e^{-\gamma_x \tau}]^2}{2\gamma_x}\sigma_x^2 \\ Cov[\Delta y, \Delta y_{-1}] & = & -\frac{[1-e^{-\gamma_y \tau}]^2}{2\gamma_y}\sigma_y^2 \\ Cov[\Delta x, \Delta y_{-1}] & = & -\frac{[1-e^{-\gamma_x \tau}]^2}{\gamma_x+\gamma_y}\sigma_x\sigma_y\rho_{xy} \\ Cov[\Delta y, \Delta x_{-1}] & = & -\frac{[1-e^{-\gamma_y \tau}]^2}{\gamma_x+\gamma_y}\sigma_x\sigma_y\rho_{xy} \end{array} \tag{14}$$

[10]Model (4) can be estimated by different methods. As the transition density functions are available in closed form, maximum likelihood estimation is feasible (see Lo (1988) for a discussion). Alternatively, GMM estimation can be applied based on conditional and unconditional moments (see e.g. Hansen and Scheinkmann (1995) and Singleton (2001)).

In general, the four moments on the right hand side of equation (13) depend non-linearly on the parameter set $\{a, b, \gamma_x, \gamma_y, \sigma_x, \sigma_y, \rho_{xy}\}$ and thus also on the parameter set $\{\alpha_p, \alpha_s, \beta_p, \beta_s, \sigma_p, \sigma_s, \rho_{ps}\}$. However, for special cases, we are able to determine the parameters a, b, γ_x, and γ_y explicitly from these moments. Since a, b, γ_x, and γ_y determine g^x, g^y, g^{xy}, and A, they suffice to construct simple correction formulae for the usual variance-covariance estimate in terms of "observable" variables.

4.1 No Feedback

The first case we study is the one with feedback parameters α_s and β_p equal to zero in model (4). This parameter restriction leads to two correlated O-U processes without feedback in the drift. We can interpret this case as a simple extension of the Lo and Wang (1995) univariate trending O-U process to a two-factor model. In terms of model (5), the restriction $\alpha_s = \beta_p = 0$ means that $a = b = 0$. Thus, the matrix A reduces to the identity matrix and the system (p, s) becomes the system (x, y). Therefore, that $\gamma_x = \alpha_p, \gamma_y = \beta_s, \sigma_x = \sigma_p, \sigma_y = \sigma_s$, and $\rho_{xy} = \rho_{ps}$ holds true.

In order to derive an adjustment to the usual variance-covariance estimate, we express the mean-reversion parameters α_p and β_s in terms of the first order return autocorrelations $\rho_p^\Delta \equiv \frac{Cov[\Delta p, \Delta p_{-1}]}{Var[\Delta p]}$ and $\rho_s^\Delta \equiv \frac{Cov[\Delta s, \Delta s_{-1}]}{Var[\Delta s]}$. From equations (9), (10), (13), and (14) it follows that $\rho_p^\Delta = -\frac{1}{2}[1 - e^{-\alpha_p \tau}]$ and $\rho_s^\Delta = -\frac{1}{2}[1 - e^{-\beta_s \tau}]$, which simplifies equation (11) to:

$$\begin{pmatrix} \sigma_p^2 \\ \sigma_s^2 \\ \rho_{ps}\sigma_p\sigma_s \end{pmatrix} = \begin{pmatrix} \frac{ln(2\rho_p^\Delta+1)}{2\rho_p^\Delta} & 0 & 0 \\ 0 & \frac{ln(2\rho_s^\Delta+1)}{2\rho_s^\Delta} & 0 \\ 0 & 0 & \frac{ln(2\rho_p^\Delta+1)+ln(2\rho_s^\Delta+1)}{2(\rho_p^\Delta+\rho_s^\Delta)} \end{pmatrix} \begin{pmatrix} \frac{Var[\Delta p]}{\tau} \\ \frac{Var[\Delta s]}{\tau} \\ \frac{Cov[\Delta p, \Delta s]}{\tau} \end{pmatrix}. \quad (15)$$

Equation (15) provides us with a simple correction formula to the usual variance-covariance estimate for autocorrelations and lead-lag patterns. The adjustment is only dependent on the asset return autocorrelations ρ_p^Δ and ρ_s^Δ. Since the cross-autocorrelations are completely specified through the autocorrelations in this case, they do not explicitly enter into the correction formula.

Of course, the adjustment for both volatility parameters σ_p and σ_s is exactly the one derived by Lo and Wang (1995).[11] The higher the autocorrelation is in absolute[12]

[11]See Lo and Wang (1995), p.95.

[12]Note that α_p and β_s must be non-negative in order to ensure that the processes are not

terms, the higher the adjustment factor. Therefore, the more negatively correlated asset returns are, the more the true value is understated by the usual variance estimate. If return autocorrelations are equal across the processes, the covariance term must be adjusted with the same factor as the variance terms. In general, however, the adjustment to the covariance term depends on both return autocorrelations and is not a monotone function of ρ_p^Δ or ρ_s^Δ.

[*Figure 2 about here.*]

Figure 2 shows the adjustment factors for the covariance term as well as for the two variance terms, as a function of the autocorrelation ρ_p^Δ. For fixed unconditional variances and covariance of finite holding-period returns, the adjustment factors show us how to scale these values in order to obtain the required instantaneous parameters. The first order autocorrelation ρ_s^Δ is chosen as -0.4, which implies a β_s of 19.3 for monthly returns. For such an autocorrelation, the usual variance estimate understates σ_s^2 by 50%, while the adjustment factor for σ_p^2 increases from 1 for an autocorrelation ρ_p^Δ of zero above all bounds when ρ_p^Δ is approaching -0.5. This course is well known from Lo and Wang (1995). However, unlike Lo and Wang, we are also interested in the estimate of the covariance term $\sigma_p\sigma_s\rho_{ps}$. As Figure 2 shows, the usual covariance estimate understates the true value by the same factor as σ_s^2 when ρ_p^Δ is zero. Increasing ρ_p^Δ leads first to a smaller adjustment factor up to a minimal value. Thereafter the adjustment increases in ρ_p^Δ, and for $\rho_p^\Delta = \rho_s^\Delta$ it again becomes equal to the factor of σ_s^2, which also holds for σ_p^2 in this case. When ρ_p^Δ increases further, the usual covariance estimate understates $\sigma_p\sigma_s\rho_{ps}$ more strongly, although the adjustment factor in this region is always below that of σ_p^2. It can be shown generally that the adjustment factor is never below one. Thus, as long as at least one of the two autocorrelations is non-zero, the usual covariance estimate always understates the true value. Furthermore, the adjustment factor for the covariance term always lies between the adjustment factors for the two variance terms. From these results we can conclude that, if the usual variance estimate understates at least one variance parameter due to autocorrelations, these problems transfer to the covariance term, even if the second variance parameter can be estimated without bias.

explosive. Therefore, ρ_p^Δ and ρ_s^Δ lie in the interval $(-\frac{1}{2}\ ,0]$ in this model.

4.2 Partial Feedback

The second case that we analyze in more detail is $\alpha_p > 0$, $\alpha_s < 0$, and $\beta_p = \beta_s = 0$. Thus, s follows a Brownian motion, while the drift of the p process has an error-correction term $-\alpha_p(p(t) - \frac{-\alpha_s}{\alpha_p} s(t))$. In terms of model (5), these restrictions transfer to $b = 0$, $\gamma_y = 0$, $\gamma_x = \alpha_p$, and $a = -\alpha_s/\alpha_p$, which shows directly that p and s are co-integrated. In this case, equation (11) reduces to:

$$\begin{pmatrix} \sigma_p^2 \\ \sigma_s^2 \\ \rho_{ps}\sigma_p\sigma_s \end{pmatrix} = \begin{pmatrix} \frac{1}{g^x} & a^2(1 - \frac{1}{g^x}) & 0 \\ 0 & 1 & 0 \\ 0 & a(1 - \frac{1}{g^x}) & \frac{1}{g^x} \end{pmatrix} \begin{pmatrix} \frac{Var[\Delta p]}{\tau} \\ \frac{Var[\Delta s]}{\tau} \\ \frac{Cov[\Delta p, \Delta s]}{\tau} \end{pmatrix}. \tag{16}$$

By stating the standardized cross-autocovariance

$$\rho^{\Delta}_{xy_{-1}} \equiv \frac{Cov[\Delta x, \Delta y_{-1}]}{Cov[\Delta x, \Delta y]} = \frac{Cov[\Delta p, \Delta s_{-1}]}{Cov[\Delta p, \Delta s] - aVar[\Delta s]} \tag{17}$$

in terms of moments of asset returns we can derive the following expressions for g^x and a:[13]

$$g^x = \frac{\rho^{\Delta}_{xy_{-1}}}{ln(\rho^{\Delta}_{xy_{-1}} + 1)} \ , \tag{18}$$

$$a \;=\; \frac{Cov[\Delta p, \Delta p_{-1}]}{Cov[\Delta p, \Delta s_{-1}]} + \sqrt{\frac{Cov[\Delta p, \Delta p_{-1}]^2}{Cov[\Delta p, \Delta s_{-1}]^2} - 2\frac{Cov[\Delta p, \Delta p_{-1}]Cov[\Delta p, \Delta s]}{Cov[\Delta p, \Delta s_{-1}]Var[\Delta s]} + \frac{Var[\Delta p]}{Var[\Delta s]}} \ . \tag{19}$$

Equation (16) shows how to adjust the usual variance-covariance estimator for autocorrelation and lead-lag patterns. Through (18) and (19) we are able to express this adjustment in terms of moments of finite holding-period returns only. First, we do not need any adjustment for the estimate of the variance σ_s^2. This is not surprising since s follows a Brownian motion. Second, to analyze the adjustment for the estimate of σ_p^2, we rearrange the correction formula to be:

$$\begin{aligned} \sigma_p^2 &= \frac{ln(\rho^{\Delta}_{xy_{-1}}+1)}{\rho^{\Delta}_{xy_{-1}}} \frac{Var[\Delta p]}{\tau} + a^2\Big(1 - \frac{ln(\rho^{\Delta}_{xy_{-1}}+1)}{\rho^{\Delta}_{xy_{-1}}}\Big)\frac{Var[\Delta s]}{\tau} \\ &= \frac{Var[\Delta p]}{\tau} + D\Big(\frac{ln(\rho^{\Delta}_{xy_{-1}}+1)}{\rho^{\Delta}_{xy_{-1}}} - 1\Big), \end{aligned} \tag{20}$$

where $D \equiv \frac{Var[\Delta p]}{\tau} - a^2\frac{Var[\Delta s]}{\tau}$ is the difference between the annualized variances of Δp and $a\Delta s$. Since $\frac{ln(\rho^{\Delta}_{xy_{-1}}+1)}{\rho^{\Delta}_{xy_{-1}}} - 1 \leq 0$, equation (20) shows that the usual estimate

[13] See Appendix 2 for a derivation.

for σ_p^2 understates the true value when $D > 0$, but overstates it when $D < 0$. The more negative $\rho_{xy_{-1}}^{\Delta}$ becomes, the more severe the estimation error. A strong lead of s, and similar values of the variance of Δs and the covariance between Δs and Δp indicate such situations.

We can derive a similar correction formula for the covariance term. Rearranging equation (16) leads to

$$\begin{aligned} \rho_{ps}\sigma_p\sigma_s &= a\Big(1 - \tfrac{ln(\rho_{xy_{-1}}^{\Delta}+1)}{\rho_{xy_{-1}}^{\Delta}}\Big)\tfrac{Var[\Delta s]}{\tau} + \tfrac{ln(\rho_{xy_{-1}}^{\Delta}+1)}{\rho_{xy_{-1}}^{\Delta}}\tfrac{Cov[\Delta p,\Delta s]}{\tau} \\ &= \tfrac{Cov[\Delta p,\Delta s]}{\tau} + \bar{D}\Big(\tfrac{ln(\rho_{xy_{-1}}^{\Delta}+1)}{\rho_{xy_{-1}}^{\Delta}} - 1\Big), \end{aligned} \tag{21}$$

where $\bar{D} \equiv \frac{Cov[\Delta p,\Delta s]}{\tau} - a\frac{Var[\Delta s]}{\tau}$. Again, the above equation shows that the usual estimate for $\rho_{ps}\sigma_p\sigma_s$ understates the true value when $\bar{D} > 0$ and overstates it when $\bar{D} < 0$. As before, the estimation error increases when the standardized cross-autocovariance $\rho_{xy_{-1}}^{\Delta}$ becomes more negative.

In contrast to the case with no feedback, an estimation error can occur even though asset returns are not autocorrelated. To see this, note that two parameter sets can lead to an autocorrelation ρ_p^{Δ} of zero. The first is $\gamma_x = 0$. This means that both processes follow a Brownian motion and therefore estimator (3) is adequate.

More interesting is the second parameter set, where ρ_p^{Δ} becomes zero also for $\gamma_x > 0$. Then, $\sigma_x = -2a\sigma_y\rho_{xy}$, or equivalently, $\sigma_p = a\sigma_s$. In this case, the usual estimate for σ_p^2 is adequate, since D in equation (20) equals zero.

However, the usual covariance estimate can seriously differ, even asymptotically, from the true parameter value. Figure 3 shows the asymptotic limit of the estimator for the correlation coefficient ρ_{ps} as a function of $\rho_{xy_{-1}}^{\Delta}$. The true parameters are $\sigma_p^2 = \sigma_s^2 = 0.3$, $a = 1$, and $\rho_{ps} = -1, -0.5, 0$, or 0.5. Thus, $\sigma_p = a\sigma_s$ and the autocorrelation ρ_p^{Δ} is equal to zero, although α_p (and therefore γ_x) is not. Since we fixed $a = 1$, we can state the adjustment for the estimate of $\sigma_p\sigma_s\rho_{ps}$ completely in terms of $\rho_{xy_{-1}}^{\Delta}$.

[Figure 3 about here.]

Due to the lead-lag patterns we obtain an asymptotic bias when estimating the instantaneous correlation via estimator (3). The stronger the lead-lag relation is, the more the estimated correlation coefficient overstates the true value. When $\rho^{\Delta}_{xy_{-1}}$ converges towards -1, which implies that the error-correction term leads to a very fast adaptation of p towards s, the estimated correlation coefficient tends towards 1, whatever the true value is. The intuition behind this result is that estimator (3) does not distinguish between price changes due to the error-correction term and those which are due to innovations according to the Brownian motion. Therefore, we erroneously interpret the error correction as a very high instantaneous correlation, although the true correlation can take any value.

4.3 Full Symmetric Feedback

The third case we consider is $\alpha_p = \beta_s = -\alpha_s = -\beta_p \equiv \alpha > 0$. This restriction leads to a symmetric, co-integrated system where both feedback parameters are non-zero. In terms of model (5) the parameter restriction implies $a = -1, b = 1, \gamma_x = 0$, and $\gamma_y = 2\alpha$. Since g^x becomes one and $g^y = g^{xy}$, equation (11) leads to

$$\begin{pmatrix} \sigma_p^2 \\ \sigma_s^2 \\ \rho_{ps}\sigma_p\sigma_s \end{pmatrix} = \frac{1}{4}\begin{pmatrix} 1+\frac{3}{g^y} & 1-\frac{1}{g^y} & 2-\frac{2}{g^y} \\ 1-\frac{1}{g^y} & 1+\frac{3}{g^y} & 2-\frac{2}{g^y} \\ 1-\frac{1}{g^y} & 1-\frac{1}{g^y} & 2+\frac{2}{g^y} \end{pmatrix}\begin{pmatrix} \frac{Var[\Delta p]}{\tau} \\ \frac{Var[\Delta s]}{\tau} \\ \frac{Cov[\Delta p,\Delta s]}{\tau} \end{pmatrix}. \tag{22}$$

Analogous to the previous case, we derive an expression for g^y,

$$g^y = \frac{\rho^{\Delta}_{yx_{-1}}}{ln(\rho^{\Delta}_{yx_{-1}}+1)}, \tag{23}$$

and state $\rho^{\Delta}_{yx_{-1}}$ in terms of the unconditional moments of the asset returns by inverting the systems (9) and (13):

$$\rho^{\Delta}_{yx_{-1}} = \frac{-Cov[\Delta p,\Delta p_{-1}] + Cov[\Delta s,\Delta s_{-1}] - Cov[\Delta p,\Delta s_{-1}] + Cov[\Delta s,\Delta p_{-1}]}{-Var[\Delta p] + Var[\Delta s]}. \tag{24}$$

Equation (22) in conjunction with equations (23) and (24) provides simple adjustments for autocorrelations and lead-lag patterns. For $\rho^{\Delta}_{yx_{-1}} = 0$ the above adjustment reduces to the identity matrix, and thus the usual variance-covariance estimator is adequate. If a lead-lag relation holds, i.e. $\rho^{\Delta}_{yx_{-1}} < 0$, the estimator gets asymptotically biased (either upwards or downwards). Both the estimated instantaneous variance and the covariance converge to the same value

$$\frac{1}{4}\frac{Var[\Delta p]}{\tau} + \frac{1}{4}\frac{Var[\Delta s]}{\tau} + \frac{1}{2}\frac{Cov[\Delta p,\Delta s]}{\tau} = \frac{Var[\Delta x]}{\tau} \tag{25}$$

when $\rho^{\Delta}_{yx_{-1}} \rightarrow -1$. It can be shown that the highest of the three values $\{\sigma_p^2, \sigma_s^2, \sigma_p\sigma_s\rho_{ps}\}$ must lie above the limiting value in equation (25). Hence, in the presence of lead-lag patterns, the usual sample variance or covariance is a downward biased estimate of this value. Analogous, the lowest of the three values $\{\sigma_p^2, \sigma_s^2, \sigma_p\sigma_s\rho_{ps}\}$ lies below the limiting value in equation (25) and therefore the usual estimate overstates the true value.

Figure 4 shows the asymptotic limits of the estimates for the variances σ_p^2 and σ_s^2, as well as the covariance $\sigma_p\sigma_s\rho_{ps}$ as a function of the first order standardized cross-autocovariance $\rho^{\Delta}_{yx_{-1}}$.

[Figure 4 about here.]

The true parameter values are $\sigma_p^2 = 0.3$, $\sigma_s^2 = 0.05$ and $\sigma_p\sigma_s\rho_{ps} = 0.0612$. For a standardized cross-autocovariance $\rho^{\Delta}_{yx_{-1}}$ of zero, estimator (3) comes up with the correct values. The stronger the lead-lag relation, the more the usual estimator understates σ_p^2 and overstates σ_p^2 and $\sigma_p\sigma_s\rho_{ps}$. For a perfect lead-lag relation ($\rho^{\Delta}_{yx_{-1}} \rightarrow -1$), all three values are estimated as $Var[\Delta x]/\tau = 0.1181$.

The analysis of the three cases presented in this section demonstrates that changes in return autocorrelations and lead-lag patterns clearly affect the instantaneous variance-covariance parameters. Next, we present an application in order to highlight the impact on derivatives whose prices depend on the covariance between two assets' returns.

5 Application: Stock Option Plans

Derivatives depending on at least two traded assets arise in various contexts. Mahoney (1995) provides several common forms of such derivatives. One prominent example is an index-linked stock option plan for executive or non-executive employees. An increasing number of firms regards stock option plans as a very flexible, tax-efficient way to reward employees for performance, and to attract a motivated staff. Usually, a stock option gives an employee the right to buy a certain number of company shares at a so-called grant price within a certain time period. In some plans, the grant price is set equal to the market price of the stock on the date the employee receives the options ($P(0)$). Other plans require that the employee benefits from the plan only if the company's share price outperforms a certain index.[14] A usual way to determine the grant price in the case of an index-linked option plan is to multiply the share price $P(T)$ at the exercise date by the ratio of one plus the percentage index performance and one plus the percentage performance of the own stock. Thus, when exercising the option in T, the grant price[15] is

$$P(T)\frac{1+\frac{S(T)-S(0)}{S(0)}}{1+\frac{P(T)-P(0)}{P(0)}} = S(T)\frac{P(0)}{S(0)}, \tag{26}$$

where $S(0)$ and $S(T)$ denote the index levels at times 0 and T, respectively. For ease of exposition, assume that the current index level equals the current stock price. In this case, the exercise value for one share is

$$\max(0, P(T) - S(T)), \tag{27}$$

which is exactly the pay-off from the option to exchange one risky asset for another.

5.1 Comparative Static Analysis

First, we provide the results of a comparative static analysis, which shows the quantitative effect of different lead-lag patterns on the value of a 'stylized' index-linked employee's stock option. In order to value such an exchange option, we employ the option pricing formula derived by Margrabe (1978). Of course, the Margrabe formula does not incorporate many features of specific stock option plans.[16] However, as our focus is on the analysis of the impact of predictability on derivatives whose prices depend on the covariance between two asset returns, it is a natural first step to gain some insight from a stylized plan with closed-form solutions. To highlight the impact of cross-autocorrelations, as opposed to autocorrelations, we chose the case of partial feedback and a zero return autocorrelation of both processes.

[14]See e.g. the stock options plans of BASF, Bayer, Henkel, Lufthansa, Münchener Rück or Vodafone, among many others.

[15]In some cases, this grant price is adjusted with some prespecified factor.

[16]Typically, stock options are American type options and the employee is required to hold the option during the so-called "freeze-out" period before exercising. In addition, dividends must be considered, as well as a possible dilution from exercising the options. Another aspect that we do not discuss here is the question of whether the stock option should be worth less than the market value for the employee, given that he might be unable to hold a well-diversified portfolio or hedge the option dynamically.

The three panels of Table 2 show prices of exchange options for times to maturity of one, five, and ten years, respectively. We fixed both the annualized finite holding-period return variances $Var[\Delta p^*]/\tau$ and $Var[\Delta s^*]/\tau$ at the same value of 0.1456. This value was chosen for comparison reasons with Table I of Lo and Wang (1995). The annualized covariance was fixed at $Cov[\Delta p^*, \Delta s^*]/\tau = 0.0728$, which implies a correlation of $\rho^{\Delta}_{ps} = 0.5$. Holding these moments fixed, we vary the cross-autocorrelation of finite holding-period returns $\rho^{\Delta}_{ps_{-1}}$ within the feasible parameter range of zero to 0.5. From equations (16),(17), (18), and (19), we then obtain the instantaneous parameters σ_p^2, σ_s^2 and $\rho_{ps}\sigma_p\sigma_s$, which serve as inputs to the pricing model by Margrabe (1978). Option prices are provided for a current share price of $P(0) = 40$ and current index levels $S(0)$ of 40, 45, or 50.

[Table 2 about here.]

We see that for all maturities and all current index levels the option price rises with the cross-autocorrelation. As we have partial feedback with zero return autocorrelations and equal variances, $a = \sqrt{Var[\Delta p^*]}/\sqrt{Var[\Delta s^*]} = 1$. In this case we know from Section 4.4.2 that both σ_p^2 and σ_s^2 will be unaffected by changes of $\rho^{\Delta}_{ps_{-1}}$. Thus, the lead of Δs^* affects option prices solely through the instantaneous correlation ρ_{ps}. The last row of the table shows the values of ρ_{ps} that correspond to different cross-autocorrelations. Though the correlation of simultaneous finite holding-period returns is as high as 0.5, the instantaneous correlation can become negative when the cross-autocorrelation exceeds values of about 0.4. When we look from the perspective of parameter estimation as in Section 4.4.2, we see again that the correlation of finite holding-period returns is an estimator of the instantaneous correlation that might not even give us the correct sign. In this context, we note that the properties of ρ^{Δ}_{ps} as an estimator of ρ_{ps} do not depend on the length of the return interval τ, provided that the annualized variances $Var[\Delta p^*]/\tau$, $Var[\Delta s^*]/\tau$ and the covariance $Cov[\Delta p^*, \Delta s^*]/\tau$ are fixed.

A time to maturity of one year allows us to compare the results with those reported by Lo and Wang (1995), Table I, p. 98, which looked at the impact of return autocorrelation on the price of vanilla call options. For at-the-money options ($S(0) = 40$), a cross-autocorrelation of 0.45 increases the price of an exchange option by more than 58%, compared with the case of zero cross-autocorrelation. Even for $\rho^{\Delta}_{ps} = 0.2$ we still have a price increase of 12%. Lo and Wang report a price increase of 50%, when the autocorrelation is -0.45, and an increase of 11% for an autocorrelation of -0.2. Therefore, the lead-lag effects are even stronger than the already strong autocorrelation effects reported by Lo and Wang. When we look at current index levels of 45 or 50, i.e. the stock has to increase more than the index to receive a payment from the option plan, the relative impact of cross-autocorrelations on option prices is even more pronounced than in the base case of at-the-money options.

The results for the more realistic maturities of five and ten years are provided in the next two panels of Table 2. In absolute terms, the longer the time to expiration, the more strongly the option prices are effected by cross-autocorrelations. This result is intuitive, since the option's "vega" increases with the expiration date. When option prices are measured in percentage points of the price that prevails under zero cross-autocorrelation, the price impact of an increasing $\rho^{\Delta}_{ps_{-1}}$ even declines slightly with time to maturity.

5.2 The SAP Long Term Incentive-Plan 2000: A Case Study

As a further illustration of our results we look at the stock option plan launched by the German software firm SAP in 2000. According to this plan, a total amount of up to 6,250,000 call options on SAP shares[17] can be distributed among certain employees. These options are American type options with a time to expiration of ten years, but there are freeze-out periods of either two, three, or four years before an option can be exercised. One option entitles the holder to buy one share for a grant price, which is equal to the then prevailing share price minus the out-performance of the SAP share over the GSTI Software Index. The exact terms lead to formula (26) for the grant price and we can therefore interpret the stock option as an option to exchange one risky asset (the index) for another (the SAP share), with a normalized index such that the index level, when converted into Euro, equals the share price at time 0. Shares needed when options are exercised can either be own shares or new shares. However, we assume that no new shares are issued, which means that we need not consider dilution effects. The stock option plan was accepted in a general meeting of the shareholders on 1/18/2000.

Imagine that, on December 31^{st} 1999, one wanted to value an option according to

[17] Originally, the options were written on preferred stock, but in 2001 SAP converted all its preferred stock to common stock.

the SAP suggestions in order to assist the shareholders with their decision on the plan and for accounting purposes.[18] Furthermore, imagine that one used daily log-returns of the SAP stock and the GSTI Software Index over the preceding 250-day period to estimate the instantaneous variances and the correlation. The first column of Table 3 shows estimation results according to estimator (3). As expected, the SAP share has a higher variance than the index ($\sigma_p^2 > \sigma_s^2$), and we find a significant positive correlation coefficient. Given the standard errors of the parameter estimates, estimation errors seem to be a minor point for a sample size of 250.

Based on these parameter estimates, option prices are provided for freeze-out periods of two, three, and four years, using the XETRA closing price of € $161,67$ of one SAP share on 12/31/99. For both the SAP share as well as the GSTI software index, a constant dividend yield of 1% p. a. is assumed. Due to the American feature of the options, option prices are obtained numerically. As Table 3 shows, longer freeze-out periods lower option values only slightly.

[Table 3 about here.]

The results in the first column of Table 3 should be interpreted with caution, as the simple parameter estimates ignore both autocorrelations and lead-lag effects of asset returns. Figure 5 presents the estimated return autocorrelations and cross-autocorrelations for the stock and the index. The horizontal lines provide the boundaries of a 95% confidence interval. We find no significant autocorrelations, but a significant positive lead of the index, i.e. we observe a correlation of 19% between stock returns and index returns lagged by one day. All other cross-autocorrelations are insignificant.

[Figure 5 about here.]

Autocorrelations and cross-autocorrelations suggest a situation of partial feedback, which we analyzed in Section 4.4.2. This finding allows us to apply the corresponding correction formulae (16). We use only significant moments for this correction,

[18]According to the International Financial Reporting Standard 2 the " fair value" of a stock option plan should either be determined from market prices or " using a valuation technique to estimate what the price of those equity instruments would have been on the measurement date in an arm´s length transaction between knowledgeable, willing parties. The valuation technique shall be consistent with generally accepted methodologies for pricing financial instruments ...".

i.e. all non-significant moments are set equal to zero. The second column of Table 3 shows the corrected variance-covariance parameter estimates and option prices. In addition, we present the corresponding drift parameters α_p and α_s.[19] Since we have no autocorrelation and no lead of the SAP stock, the variances σ_p^2 and σ_s^2 remain unchanged compared to the results of estimator (3). But the instantaneous correlation ρ_{ps} decreases substantially from 0.315 to 0.19. This bias correction is much larger than the average sampling error, which is quantified by the standard error of 0.054. Moreover, the bias does not even disappear asymptotically with an increasing number of daily observations. Due to the lower instantaneous correlation, option prices increase by about 6.7% from almost € 92 to almost € 98.

In summary, the results of the comparative static analysis and the SAP example show that substantial errors might occur when we ignore lead-lag effects in the assessment of a stock option plan. In the example of the SAP plan, such errors are easily detected and corrected by means of sample moments.

6 Summary and Conclusion

Lo and Wang (1995) have shown that the predictability of an asset's return has an impact on derivative prices, since, for a fixed variance of finite-holding period returns, predictability changes the instantaneous variance. We extend Lo and Wang's analysis by considering derivatives that also depend on the covariance of two assets' returns and by looking at lead-lag effects as a frequently observed form of return predictability.

As a reference point for our analysis, we propose a bivariate linear diffusion model which generates a variety of lead-lag patterns. The model allows for general feedback in the drift terms of the two processes. We suggest a state-space representation, which shows how the impact of lead-lag patterns on instantaneous parameters is connected to time series properties like stationarity, non-stationarity, and co-integration.

Based on our two-factor diffusion model, we establish the link between the annualized variance-covariance matrix of finite holding-period returns and the instantaneous variance-covariance matrix. The former will, in general, be a biased estimate

[19]As our correction formulae build on a closed form method of moments estimator, we obtain estimates of all free model parameters and can easily calculate standard errors.

of the latter. In particular, the correlation between finite holding-period returns might not even provide the right sign of the instantaneous correlation. To allow for the effects of return predictability, we derive adjustments to the annualized variance-covariance matrix of finite holding-period returns. These adjustments only depend on return variances, covariances, autocovariances, and cross-autocovariances and are therefore easy to implement.

The link between the finite-holding period and instantaneous variances and covariances is also important for another application. Prices of derivatives have frequently been used to extract implied volatilities in order to forecast future volatilities. This idea has been extended to correlation forecasts by Siegel (1997), Campa and Chang (1998), Bodurtha and Shen (1999), and Walter and Lopez (2000). Our approach might help to explain why this approach is successful in some markets and fails in others (e.g. see the results of Walter and Lopez (2000)).

Our results imply that lead-lag effects can have a strong impact on correlation dependent derivatives. As an illustration, we have applied our results to the valuation of index-linked stock option plans. We have found that for realistic parameter values, an increase of the first order cross-autocorrelation from zero to 0.45 increases the option price by more than 50%. This lead-lag effect is even stronger than the autocorrelation effect documented by Lo and Wang (1995). As a concrete example, we look at the SAP Long Term Incentive-Plan 2000. The observed lead of the benchmark index can be easily captured by one of the derived correction formulae. This correction leads to a substantial decrease of the estimated instantaneous covariance and an increase of the option price. In conclusion, we believe that a consideration of lead-lag patterns is relevant for many markets and many different derivatives.

Appendix 1

In order to see how the restrictions $\gamma_x \geq 0$ and $\gamma_y \geq 0$ translate into restrictions on $\alpha_p, \alpha_s, \beta_p$, and β_s, we express γ_x and γ_y as functions of $\alpha_p, \alpha_s, \beta_p$, and β_s. Using matrix notation, we equate the drift rates of $p(t)$ and $s(t)$ given by the two parameterizations (4) and (5) and obtain

$$\begin{pmatrix} -\alpha_p & -\alpha_s \\ -\beta_p & -\beta_s \end{pmatrix} \begin{pmatrix} p(t) \\ s(t) \end{pmatrix} = \begin{pmatrix} 1 & a \\ b & 1 \end{pmatrix} \begin{pmatrix} -\gamma_x & 0 \\ 0 & -\gamma_y \end{pmatrix} \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}. \tag{28}$$

Substitution of the expressions on the right hand side of system (5) for $p(t)$ and $s(t)$ on the left hand side of equation (28) leads to

$$\begin{pmatrix} -\alpha_p & -\alpha_s \\ -\beta_p & -\beta_s \end{pmatrix}\begin{pmatrix} 1 & a \\ b & 1 \end{pmatrix} = \begin{pmatrix} 1 & a \\ b & 1 \end{pmatrix}\begin{pmatrix} -\gamma_x & 0 \\ 0 & -\gamma_y \end{pmatrix}. \quad (29)$$

Equation (29) is a non-linear system of equations we have to solve for a, b, γ_x, and γ_y, which can be equivalently written as follows:

$$\begin{aligned} -\alpha_p - b\alpha_s &= -\gamma_x && (30)\\ \beta_p + b(\beta_s - \alpha_p) - b^2\alpha_s &= 0 && (31)\\ -\beta_s - a\beta_p &= -\gamma_y && (32)\\ \alpha_s + a(\alpha_p - \beta_s) - a^2\beta_p &= 0 && (33) \end{aligned}$$

From equations (30) to (33) we obtain four solutions $(b_1, \gamma_{x_1}, a_1, \gamma_{y_1})$, $(b_1, \gamma_{x_1}, a_2, \gamma_{y_2})$, $(b_2, \gamma_{x_2}, a_1, \gamma_{y_1})$, and $(b_2, \gamma_{x_2}, a_2, \gamma_{y_2})$, with $b_{1,2}, \gamma_{x_{1,2}}, a_{1,2}, \gamma_{y_{1,2}}$ given below for the case of $\alpha_s \neq 0$, $\beta_p \neq 0$, and $(\alpha_p - \beta_s)^2 + 4\beta_p\alpha_s \geq 0$:

$$\begin{aligned} b_{1,2} &= \frac{\beta_s - \alpha_p}{2\alpha_s} \pm \sqrt{\frac{(\beta_s - \alpha_p)^2}{4\alpha_s^2} + \frac{\beta_p}{\alpha_s}} && (34)\\ -\gamma_{x_{1,2}} &= \frac{-(\alpha_p + \beta_s)}{2} \pm \sqrt{\frac{(\beta_s - \alpha_p)^2}{4} + \alpha_s\beta_p} && (35)\\ a_{1,2} &= \frac{\alpha_p - \beta_s}{2\beta_p} \pm \sqrt{\frac{(\alpha_p - \beta_s)^2}{4\beta_p^2} + \frac{\alpha_s}{\beta_p}} && (36)\\ -\gamma_{y_{1,2}} &= \frac{-(\alpha_p + \beta_s)}{2} \pm \sqrt{\frac{(\alpha_p - \beta_s)^2}{4} + \alpha_s\beta_p}. && (37) \end{aligned}$$

Not all four solutions are appropriate in the sense that they lead to a useful model. First, consider the case $\alpha_s\beta_p > 0$. In this case we obtain

$$\begin{aligned} 4a_1b_1\alpha_s\beta_p &= [(\beta_s - \alpha_p) + \sqrt{(\beta_s - \alpha_p)^2 + 4\alpha_s\beta_p}][(\alpha_p - \beta_s) + \sqrt{(\beta_s - \alpha_p)^2 + 4\alpha_s\beta_p}]\\ &= (\beta_s - \alpha_p)(\alpha_p - \beta_s) + (\beta_s - \alpha_p)^2 + 4\alpha_s\beta_p\\ &= 4\alpha_s\beta_p\ , \end{aligned}$$

which implies $a_1b_1 = 1$. Similar calculations show that $a_2b_2 = 1$. Thus, only the solutions $(b_1, \gamma_{x_1}, a_2, \gamma_{y_2})$ and $(b_2, \gamma_{x_2}, a_1, \gamma_{y_1})$ correspond to a non-degenerate case. For these solutions it holds that $\min[2\gamma_{x_1}, 2\gamma_{y_2}] = \min[2\gamma_{x_2}, 2\gamma_{y_1}] = (\alpha_p + \beta_s) - \sqrt{(\alpha_p - \beta_s)^2 + 4\alpha_s\beta_p}$.

Second, when $\alpha_s\beta_p < 0$, we find that $a_1b_2 = a_2b_1 = 1$. Consequently, we have to consider only the solutions $(\beta_1, \gamma_{x_1}, \alpha_1, \gamma_{y_1})$ and $(\beta_2, \gamma_{x_2}, \alpha_2, \gamma_{y_2})$. Since one of the parameters α_s and β_p is positive and one is negative, opposite signs in front of the square root terms in equations (35) and (37) result. This implies that $\min[2\gamma_{x_1}, 2\gamma_{y_1}] = \min[2\gamma_{x_2}, 2\gamma_{y_2}] = (\alpha_p + \beta_s) - \sqrt{(\alpha_p - \beta_s)^2 + 4\alpha_s\beta_p}$.

Therefore, we have shown that for both γ_x and γ_y to be non-negative we require

$$(\alpha_p + \beta_s) - \sqrt{(\alpha_p - \beta_s)^2 + 4\alpha_s\beta_p} \geq 0. \tag{38}$$

To complete the proof, we note that the same condition (38) must hold when either one or both parameters α_s or β_p are zero.

Appendix 2

To express the adjustment function (16) in terms of moments of finite holding-period returns, we only need expressions for a and g^x. First, from the moments in equations (10) and (14) we obtain:

$$\rho^{\Delta}_{xy_{-1}} \equiv \frac{Cov[\Delta x, \Delta y_{-1}]}{Cov[\Delta x, \Delta y]} = -(1 - e^{-\gamma_x\tau}) \tag{39}$$

and

$$\rho^{\Delta}_{x} \equiv \frac{Cov[\Delta x, \Delta x_{-1}]}{Var[\Delta x]} = -\frac{(1 - e^{-\gamma_x\tau})}{2}. \tag{40}$$

Therefore,

$$\gamma_x = -\frac{ln(\rho^{\Delta}_{xy_{-1}} + 1)}{\tau} \tag{41}$$

and

$$g^x = \frac{\rho^{\Delta}_{xy_{-1}}}{ln(\rho^{\Delta}_{xy_{-1}} + 1)}\ , \tag{42}$$

which yields to

$$\begin{pmatrix} \sigma_p^2 \\ \sigma_s^2 \\ \rho_{ps}\sigma_p\sigma_s \end{pmatrix} = \begin{pmatrix} \frac{ln(\rho^{\Delta}_{xy_{-1}}+1)}{\rho^{\Delta}_{xy_{-1}}} & a^2(1 - \frac{ln(\rho^{\Delta}_{xy_{-1}}+1)}{\rho^{\Delta}_{xy_{-1}}}) & 0 \\ 0 & 1 & 0 \\ 0 & a(1 - \frac{ln(\rho^{\Delta}_{xy_{-1}}+1)}{\rho^{\Delta}_{xy_{-1}}}) & \frac{ln(\rho^{\Delta}_{xy_{-1}}+1)}{\rho^{\Delta}_{xy_{-1}}} \end{pmatrix} \begin{pmatrix} \frac{Var[\Delta p]}{\tau} \\ \frac{Var[\Delta s]}{\tau} \\ \frac{Cov[\Delta p, \Delta s]}{\tau} \end{pmatrix}. \tag{43}$$

Second, by stating the first order standardized cross-autocovariance

$$\rho^{\Delta}_{xy_{-1}} = \frac{Cov[\Delta p, \Delta s_{-1}]}{Cov[\Delta p, \Delta s] - aVar[\Delta s]} \tag{44}$$

as well as the first order autocorrelation

$$\rho^{\Delta}_{x} = \frac{Cov[\Delta p, \Delta p_{-1}] - aCov[\Delta p, \Delta s_{-1}]}{Var[\Delta p] + a^2 Var[\Delta s] - 2aCov[\Delta p, \Delta s]} \tag{45}$$

in terms of moments of asset returns by inverting the systems (9) and (13), we obtain the following quadratic equation in a from equations (39) and (40):

$$\frac{Cov[\Delta p, \Delta s_{-1}]}{Cov[\Delta p, \Delta s] - aVar[\Delta s]} = 2\frac{Cov[\Delta p, \Delta p_{-1}] - aCov[\Delta p, \Delta s_{-1}]}{Var[\Delta p] + a^2 Var[\Delta s] - 2aCov[\Delta p, \Delta s]}. \tag{46}$$

Within our model, equation (46) has the unique[20] solution

$$a = \frac{Cov[\Delta p,\Delta p_{-1}]}{Cov[\Delta p,\Delta s_{-1}]} + \sqrt{\frac{Cov[\Delta p,\Delta p_{-1}]^2}{Cov[\Delta p,\Delta s_{-1}]^2} - 2\frac{Cov[\Delta p,\Delta p_{-1}]Cov[\Delta p,\Delta s]}{Cov[\Delta p,\Delta s_{-1}]Var[\Delta s]} + \frac{Var[\Delta p]}{Var[\Delta s]}}\ . \tag{47}$$

[20]The solution with the negative root is inconsistent with $\gamma_x > 0$.

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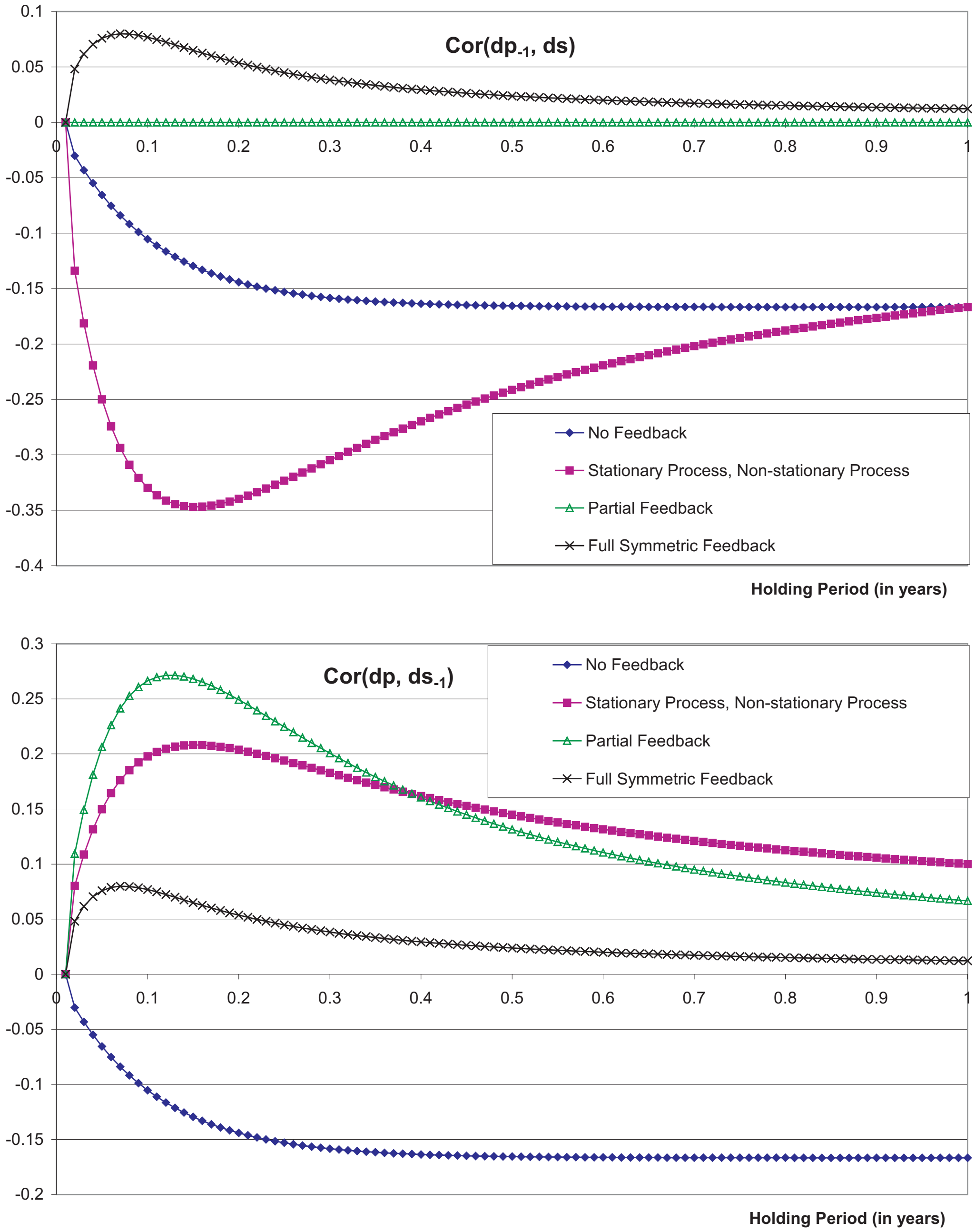

Figure 1: Return Cross-autocorrelations.

This figure shows four examples of first order return cross-autocorrelations as functions of the holding period τ, measured in years. All examples use the common parameter values $\sigma_p^2 = \sigma_s^2 = 0.3$ and $\rho_{ps} = 1/3$. The drift parameters α_p, α_s, β_p, and β_s take either the values 10, -10, or 0. The upper half of the figure depicts $\rho^{\Delta}_{p_{-1}s}$, the lower half $\rho^{\Delta}_{ps_{-1}}$.

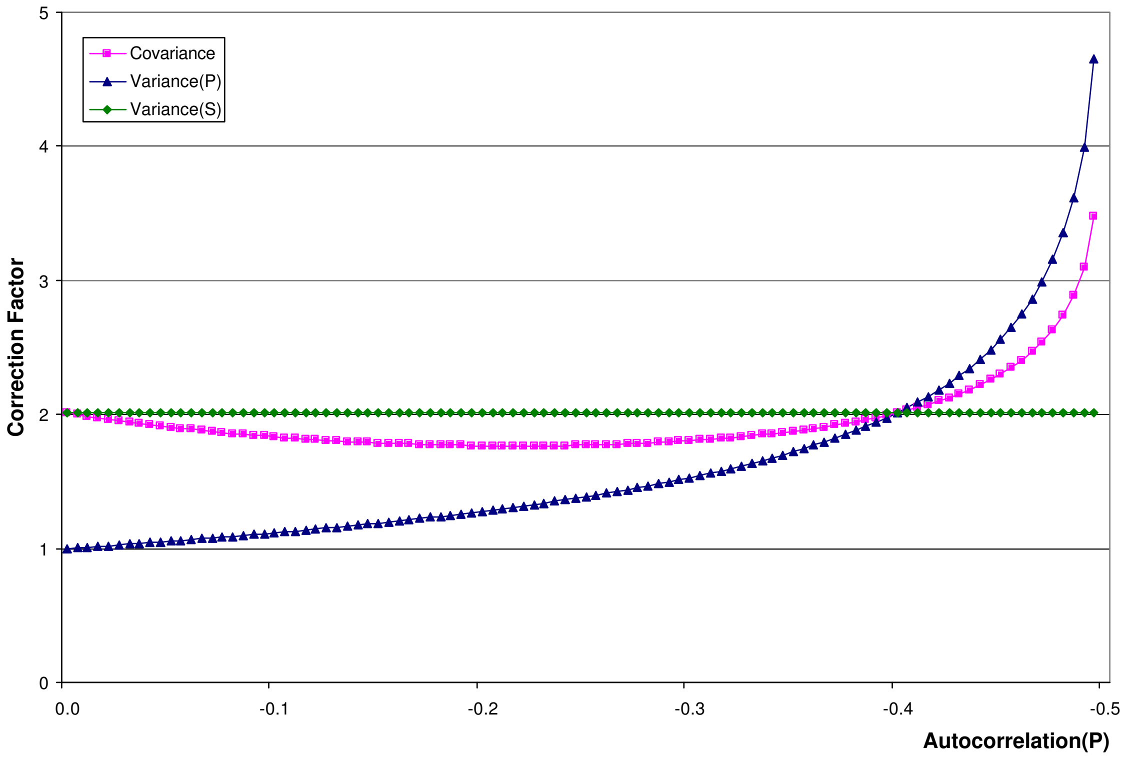

Figure 2: Adjustment Factors in the Case Without Feedback.

This figure shows the adjustment factors for the usual variance-covariance estimate under two correlated O-U processes, as a function of the first order autocorrelation ρ_p^{Δ}. The first order autocorrelation ρ_s^{Δ} is -0.4.

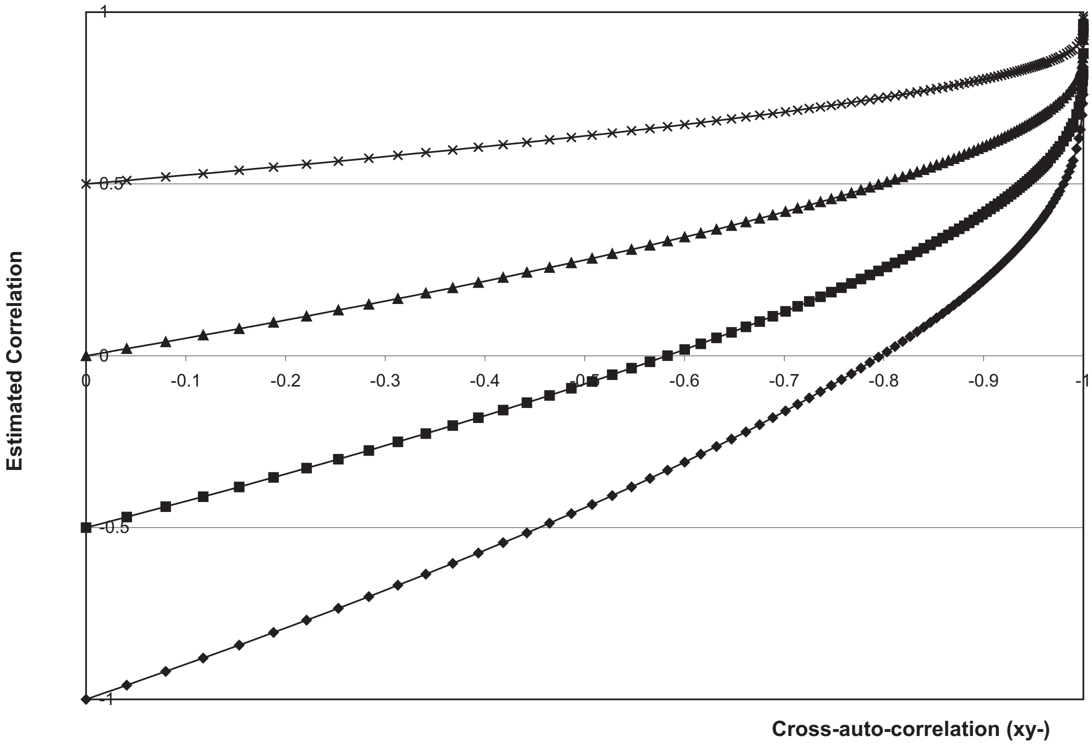

Figure 3: Estimated Correlation ρ_{ps}.

This figure shows the asymptotic limit of the usual estimator for the correlation coefficient ρ_{ps} under partial feedback as a function of $\rho^{\Delta}_{xy_{-1}}$. The chosen parameter values are $\sigma_p^2 = \sigma_s^2 = 0.3$, $a = 1$, and ρ_{ps} is either $-1, -0.5, 0$, or 0.5.

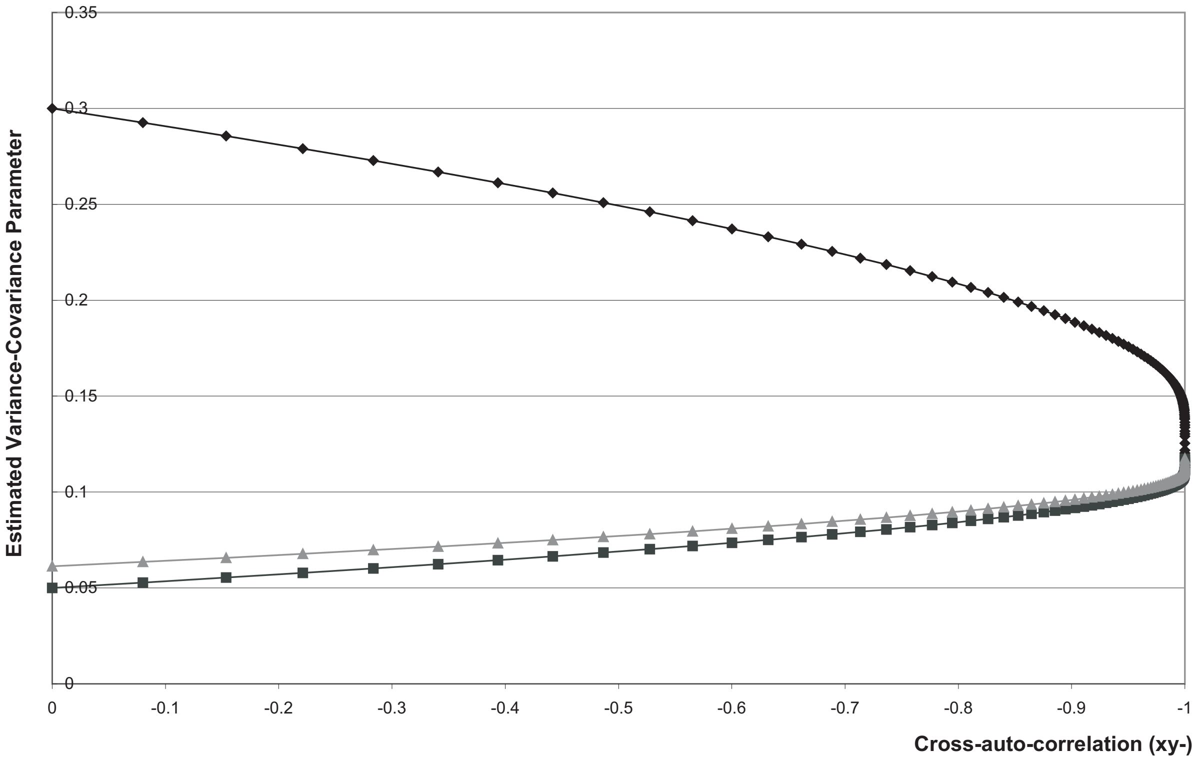

Figure 4: Estimated Variance-covariance Parameters.

This figure shows the asymptotic limits of the estimates for the variances σ_p^2 and σ_s^2 as well as the covariance $\sigma_p \sigma_s \rho_{ps}$ as a function of the first order standardized cross-autocovariance $\rho^{\Delta}_{yx_{-1}}$. The chosen parameter values are $\sigma_p^2 = 0.3$, $\sigma_s^2 = 0.05$ and $\rho_{ps} = 0.5$.

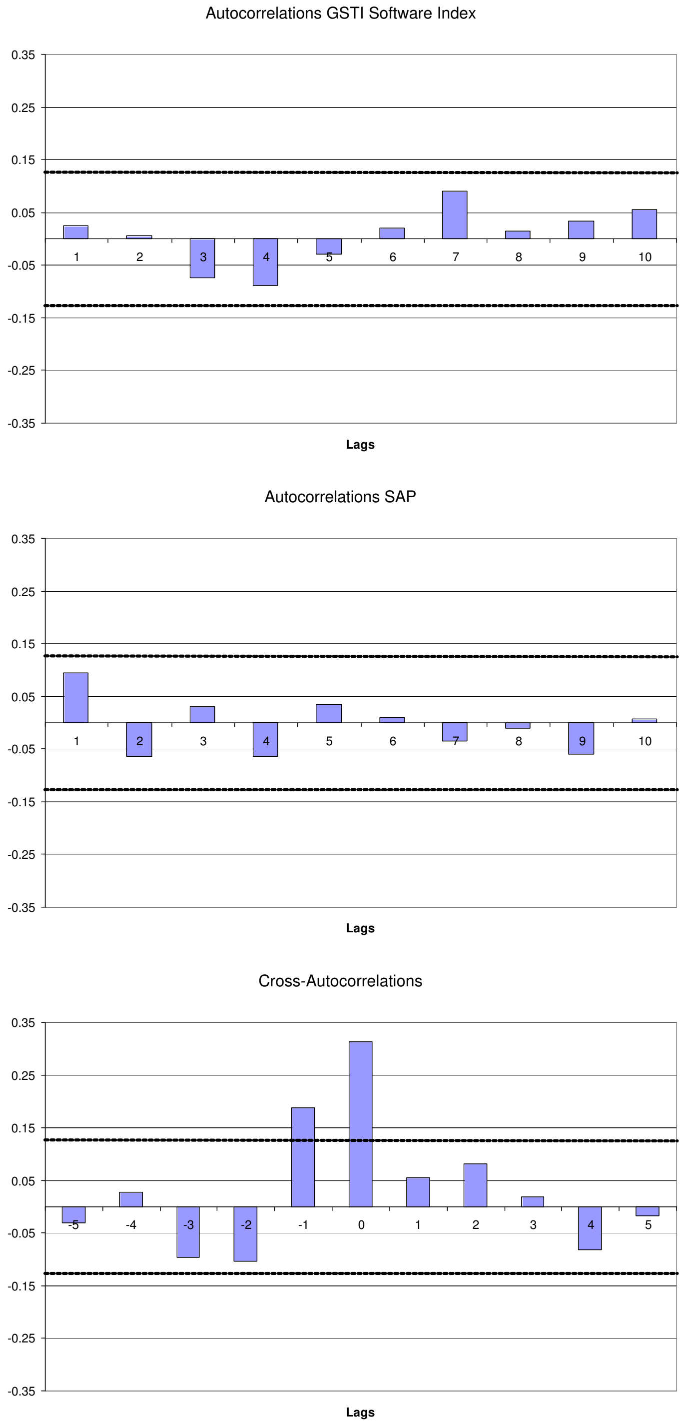

Figure 5: Daily Return Autocorrelations and Cross-Autocorrelations of SAP Stocks and the GSTI Software Index.

This figure shows estimated autocorrelations and cross-autocorrelations based on 250 daily observations from the period preceding 12/31/99. Autocorrelations are provided for up to ten lags and cross-autocorrelations for up to five lags. Negative lags for the cross-autocorrelatons refer to lagged returns of the GSTI Software Index and positive lags to lagged returns of SAP stocks. The horizontal lines are the boundaries of a 95% confidence interval.

Table 1: Instantaneous Variance-covariance Parameters and Price of an Exchange Option for Different Drift Specifications.

This table shows $\sigma_p^2, \sigma_s^2, \sigma_p\sigma_s\rho_{ps}, \rho_{ps}$, and the price of an option to exchange one asset for another. τ equals 1/12. The return variances are 0.3 and the covariance is 0.1. Option prices are calculated according to the model by Margrabe (instantaneous variance-covariance parameters, asset prices of $P = S = 40$, and a time to expiration of one year.

	Bivariate Brownian Motion $\alpha_p = 0$ $a = 0$ $\alpha_s = 0$ $b = 0$ $\beta_p = 0$ $\gamma_x = 0$ $\beta_s = 0$ $\gamma_y = 0$	No Feedback $\alpha_p = 10$ $a = 0$ $\alpha_s = 0$ $b = 0$ $\beta_p = 0$ $\gamma_x = 10$ $\beta_s = 10$ $\gamma_y = 10$	O-U Process Non-stat. Process $\alpha_p = 0$ $a = -1$ $\alpha_s = -10$ $b = 0$ $\beta_p = 0$ $\gamma_x = 0$ $\beta_s = 10$ $\gamma_y = 10$	Partial Feedback $\alpha_p = 10$ $a = 1$ $\alpha_s = -10$ $b = 0$ $\beta_p = 0$ $\gamma_x = 1$ $\beta_s = 0$ $\gamma_y =$
σ_p^2	0.3	0.442	0.063	0.3
σ_s^2	0.3	0.442	0.442	0.3
$\sigma_s\sigma_p\rho_{ps}$	0.1	0.147	0.147	0.005
ρ_{ps}	0.333	0.333	0.880	0.017
Option Price	9.927	11.959	7.271	11.959

Table 2: Option Prices for Different Cross-autocorrelations

This table shows option prices according to the model by Margrabe (1978) as functions of the finite holding-period return cross-autocorrelation $\rho^{\Delta}_{ps_{-1}}$. The current share price is always $P(0) = 40$, the current index level $S(0)$ is either 40, 45, or 50, and the time to maturity is either one year, five years, or ten years. Variances and covariance of finite holding-period returns are fixed at $Var[\Delta p^*]/\tau = 0.1456$, $Var[\Delta s^*]/\tau = 0.1456$, and $Cov[\Delta p^*, \Delta s^*]/\tau = 0.0728$.

$S(0)$	$\rho^{\Delta}_{ps_{-1}}$						
	0	0.05	0.1	0.2	0.3	0.4	0.45
Time to maturity: One year							
40	6.052	6.210	6.388	6.828	7.456	8.532	9.590
45	4.230	4.391	4.571	5.019	5.663	6.774	7.873
50	2.922	3.071	3.241	3.667	4.288	5.380	6.477
Time to maturity: Five years							
40	13.214	13.542	13.910	14.811	16.078	18.200	20.200
45	11.665	12.010	12.396	13.345	14.680	16.913	19.031
50	10.338	10.693	11.092	12.071	13.453	15.775	17.982
Time to maturity: Ten years							
40	18.148	18.571	19.042	20.185	21.763	24.314	26.619
45	16.864	17.310	17.808	19.015	20.683	23.382	25.821
50	15.723	16.188	16.707	17.966	19.709	22.534	25.090
ρ_{ps}	0.5	0.473	0.442	0.361	0.236	-0.006	-0.279

Table 3: Parameter Estimates and Theoretical Values of SAP Stock Options

This table shows parameter estimates resulting from estimator (3), corrected estimates obtained from equations (16) and the corresponding theoretical prices (in €) of one SAP stock option with ten years to maturity on 12/31/99, when the stock price was €161, 67. Option prices are given for freeze-out periods of 2, 3, and 4 years.

	Simple Parameter Estimates: Equations (3) (standard errors)	Corrected Parameter Estimates: Equations (16) (standard errors)
σ_p^2	0.253 (0.034)	0.253 (0.034)
σ_s^2	0.166 (0.016)	0.166 (0.016)
ρ_{ps}	0.315 (0.054)	0.190 (0.074)
Option Price 2 years	91.80	97.94
Option Price 3 years	91.78	97.92
Option Price 4 years	91.71	97.84
α_p	0	86.734 (32.523)
α_s	0	-107.136 (43.030)
β_p	0	0
β_s	0	0