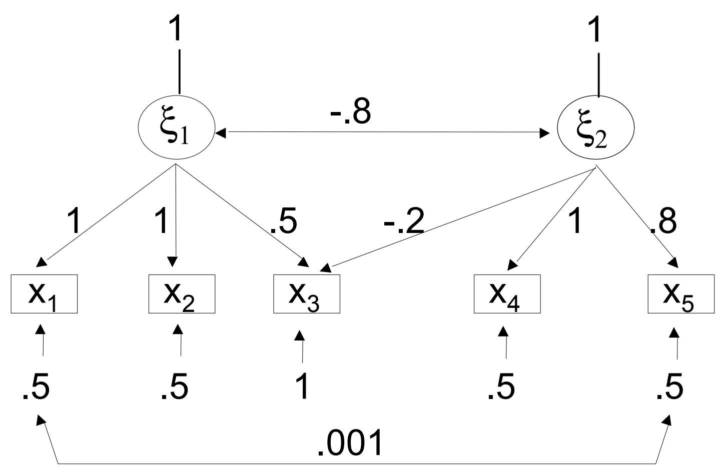
Simulation and Bootstrapping

Simulation and Bootstrapping are becoming more and more important methods to investigate statistical models or to make statistical inferences. Using interactive LISREL both bootstrapping and simulation can be done.

Simulation

In Monte-Carlo simulations data are generated using pseudo random-generators. As a simple example the following structure will be investigated:



For this structure samples of realizations can be generated with the random generator implemented in PRELIS. The simulation study is done in 5 steps.

STEP 1:
Compute the implied variances and covariances of the observed variables by a LISREL model with fixed parameters only:

```
Computing implied variances and covariances of observed variables
from a known structure
DA NI=5 NO=1 NG=1 MA=CM
CM
1 0 1 2*0 1 3*0 1 4*0 1 /
LA
X1 X2 X3 X4 X5
MO NX=5 NK=2 LX=FU,FI PH=SY,FI TD=SY,FI
LK
Xi1 Xi2
VA 1.0 PH(1,1) PH(2,2) LX(1,1) LX(2,1) LX(4,2) TD(3,3)
VA -0.8 PH(2,1)
VA 0.8 LX(5,2)
VA 0.5 LX(3,1) TD(1,1) TD(2,2) TD(4,4) TD(5,5)
VA -0.2 LX(3,2)
VA 0.001 TD(5,1)
OU  ND=6 IT=0 RS SI=simulat1.sis
```

All parameter values are are fixed to the values shown in the path diagram. On the ou-command SI=filename is used to write the implied covariance matrix in free format to a file.

STEP 2:
Compute a Cholesky decomposition of the variance/covariance-matrix:

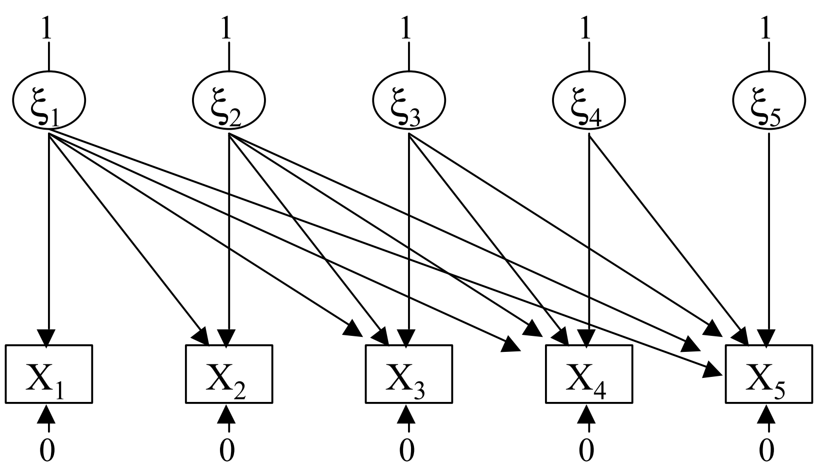
In a Cholesky decomposition any symmetric positive-definite matrix **S** is decomposed in two triagonal matrices **C** and $\mathbf{C}^{/}$:

$$\mathbf{S} = \mathbf{C} \times \mathbf{C}^{/}$$

If one is thinking of **S** as a variance/covariance matrix and **C** as a triangular matrix of factor loadings, this decomposition can be seen as a CFA, where k indicators $\mathbf{x_k}$ are explained by k uncorrelated factors :

$$\mathbf{x} = \mathrm{L}^{x}\mathrm{x} + \mathbf{0} \text{ with } \mathrm{F} = \mathbf{I} \quad \text{Þ} \quad \mathrm{S}_{\mathbf{xx}'} = \mathrm{L\,F\,L}' + \mathbf{0} = \mathrm{L\,L}'$$

For the example the Cholesky decomposition is given in the following path diagram:



Note: The only free parameters are the factor loadings, that gives the cells of the Cholesky decomposition.

For the example LISREL computes the Choleski-decomposition:

```
Choleski-decomposition of implied variances and covariances
DA NI=5 NO=100  MA=CM
LA
x1 x2 x3 x4 x5
CM FI=SIMULAT1.SIS
MO NX=5 NK=5 LX=FU,FI PH=ID TD=ZE
LK
Xi1 Xi2 Xi3 Xi4 Xi5
FR LX(1,1) LX(2,1) LX(2,2) LX(3,1) LX(3,2) LX(3,3) LX(4,1) LX(4,2)
FR LX(4,3) LX(4,4) LX(5,1) LX(5,2) LX(5,3) LX(5,4) LX(5,5)
OU ME=ML ND=6 IT=250
```

Result:

```
LAMBDA-X
          Xi1        Xi2        Xi3        Xi4        Xi5
     --------   --------   --------   --------   --------
x1   1.224745
x2   0.816497  -0.912871
x3   0.538888  -0.240998  -1.049533
x4  -0.653198   0.292119   0.169218   0.979472
x5  -0.521742   0.234425   0.135626   0.375477   0.808365
```

STEP 3:
Use the Cholesky decomposition to generate the covariance structure from uncorrelated random numbers with PRELIS2.

The results of the Cholesky decomposition can be used to generate samples of the variance/ covariance-matrix based on NO number of cases (here: 250)
The „new“-command generates the new variables.
With „RP=100“ 100 replications (samples) are generated. All 100 covariance matrices are stored in the file „simulat.cm“.

```
Generating simulated data
DA NO=250 RP=100
new v1=NRAND
new v2=NRAND
new v3=NRAND
new v4=NRAND
new v5=NRAND
new x1= 1.224745*v1
new x2= 0.816497*v1-0.912871*v2
new x3= 0.538888*v1-0.240998*v2-1.049533*v3
new x4=-0.653198*v1+0.292119*v2+0.169218*v3+0.979472*v4
new x5=-0.521742*v1+0.234425*v2+0.135626*v3+0.375477*v4+0.808365*v5
sd v1-v5
CO ALL
OU cm=simulat.cm ND=6 XM IX=123456789 XO
```

ix= number starts the random generator with known number

Output: XO on the ou-command forces to print output only for the first sample

```
Total Sample Size =    250
Univariate Summary Statistics for Continuous Variables
Variable      Mean  St. Dev.   T-Value  Skewness  Kurtosis  Minimum Freq.  Maximum Freq.
--------      ----  --------   -------  --------  --------  ------- -----  ------- -----
      x1   -0.020     1.273    -0.248    -0.235     0.187   -3.557     1    3.304     1
      x2   -0.048     1.321    -0.575    -0.147    -0.001   -3.584     1    3.260     1
      x3   -0.012     1.205    -0.162    -0.055    -0.513   -3.340     1    3.000     1
      x4   -0.027     1.267    -0.335     0.008    -0.493   -2.990     1    2.851     1
      x5    0.009     1.047     0.130    -0.058     0.197   -3.128     1    2.672     1
...
Covariance Matrix
               x1          x2          x3          x4          x5
         --------    --------    --------    --------    --------
      x1    1.620
      x2    1.115       1.746
      x3    0.775       0.811       1.451
      x4   -0.875      -0.971      -0.657       1.605
      x5   -0.633      -0.685      -0.513       0.803       1.097
...

 0.16203D+01  0.11151D+01  0.17462D+01  0.77493D+00  0.81119D+00  0.14514D+01
-0.87543D+00 -0.97149D+00 -0.65732D+00  0.16048D+01 -0.63295D+00 -0.68535D+00
-0.51311D+00  0.80329D+00  0.10968D+01
 0.15488D+01  0.10369D+01  0.14144D+01  0.63419D+00  0.43879D+00  0.12680D+01
-0.77979D+00 -0.66971D+00 -0.55835D+00  0.14956D+01 -0.70532D+00 -0.59937D+00
-0.45123D+00  0.74072D+00  0.11377D+01
...
 0.14861D+01  0.95709D+00  0.14583D+01  0.83406D+00  0.77928D+00  0.16800D+01
-0.75690D+00 -0.77827D+00 -0.81205D+00  0.16094D+01 -0.64809D+00 -0.63602D+00
-0.65927D+00  0.10076D+01  0.12542D+01
```

1. sample

2. sample

100. sample

STEP 4:
Estimate the parameters in each of the generated samples:

```
Analyze simulated covariance matrices
DA NI=5 NO=250 MA=CM RP=100
CM=simulat.cm
LA
X1 X2 X3 X4 X5
MO NX=5 NK=2 LX=FU,FI PH=SY,FR TD=SY
LK
Xi1 Xi2
VA 1.0  LX(1,1) LX(4,2)
FR LX(2,1) LX(3,1) LX(3,2) LX(5,2) TD(5,1)
OU  ND=3 XO=2 PV=simulat.par SV=simulat.se GF=simulat.gf
```

The RP-option on the DA-command forces LISREL to read in 100 covariance matrices from the file “simulat.cm” and estimate the same same model for each of them.
XO=2 forces to print only the first 2 outputs.
The estimated parameters are stored by PV=simulat.par in the file”simulat.par”, the estimated standard errors are stored by SC=simulat.se in the file “simulat.se”, and the goodness-of-fit statistics are store by GF=simulat.gf in the file “simulat.gf”.

output:

```
...
Parameter Specifications
        LAMBDA-X
                Xi1         Xi2
           --------    --------
      X1         0           0
      X2         1           0
      X3         2           3
      X4         0           0
      X5         0           4
        PHI
                Xi1         Xi2
           --------    --------
     Xi1         5
     Xi2         6           7
        THETA-DELTA
                 X1          X2          X3          X4          X5
           --------    --------    --------    --------    --------
      X1         8
      X2         0           9
      X3         0           0          10
      X4         0           0           0          11
      X5        12           0           0           0          13
...
```

Generated files:

simulat.par

```
    1   0   0
0.107834D+01 0.685476D+00-0.606643D-01 0.726767D+00 0.103395D+01-0.885357D+00
0.110512D+01 0.586334D+00 0.543898D+00 0.887871D+00 0.499676D+00 0.687219D-02
0.513014D+00
    2   0   0
0.809629D+00 0.195412D+00-0.451149D+00 0.888602D+00 0.128117D+01-0.802121D+00
0.832879D+00 0.267897D+00 0.574595D+00 0.908127D+00 0.662721D+00 0.109870D-01
0.479861D+00
...
  100   0   0
0.978741D+00 0.626399D+00-0.261804D+00 0.819185D+00 0.977706D+00-0.777105D+00
0.123090D+01 0.508303D+00 0.521723D+00 0.957123D+00 0.378497D+00-0.192359D-01
0.428848D+00
```

For each repetition:

1. line: 1) number of repetition
   2) convergence: 0=yes; 1=no;
   2=yes, but significance p<.0005 or p>.9995 and confidence limits for some fit statistics are not computed)
   3) admissability: 0=yes; 1=no
2. line: 1) first estimated parameter (here: lx(2,1))
   2) second estimated parameter (here: lx(3,1))
   ...
4. line: 1) 13. estimated parameter (here: td(5,5))

Generated files:

simulat.se

```
    1   0   0
0.889387D-01 0.187375D+00 0.179262D+00 0.737530D-01 0.149413D+00 0.114635D+00
0.163315D+00 0.822184D-01 0.881888D-01 0.913415D-01 0.999836D-01 0.521451D-01
0.659217D-01
    2   0   0
0.744311D-01 0.130976D+00 0.172726D+00 0.101275D+00 0.166263D+00 0.108331D+00
0.144241D+00 0.975258D-01 0.804936D-01 0.881952D-01 0.994702D-01 0.564134D-01
0.763634D-01
...
  100   0   0
0.883092D-01 0.132120D+00 0.111284D+00 0.723994D-01 0.142863D+00 0.107467D+00
0.165661D+00 0.826418D-01 0.810089D-01 0.971350D-01 0.945529D-01 0.467253D-01
0.705656D-01
```

For each repetition:

1. line: 1) number of repetition ; 2) convergence ; 3) admissability
2. line: 1) standard error of first estimated parameter (here: lx(2,1))
   2) standard error of second estimated parameter (here: lx(3,1)) ...
   ...

Generated files:

simulat.gf

```
   1  0  0    2  0.92067D+00  0.63107D+00  0.93205D+00  0.62749D+00  0.00000D+00
 0.10000D+01  0.00000D+00  0.10000D+01  0.00000D+00  0.00000D+00  0.50187D+01
 0.36975D-02  0.00000D+00  0.00000D+00  0.20156D-01  0.00000D+00  0.00000D+00
 0.10039D+00  0.77209D+00  0.11245D+00  0.11245D+00  0.13261D+00  0.12048D+00
 0.28143D+01  0.69077D+03  0.70077D+03  0.26932D+02  0.30000D+02  0.72337D+03
 0.85711D+02  0.97822D+02  0.10204D-01  0.71061D-02  0.99850D+00  0.98879D+00
 0.13313D+00  0.99867D+00  0.10079D+01  0.19973D+00  0.10000D+01  0.10016D+01
 0.99334D+00  0.24922D+04
   2  0  0    2  0.36481D+01  0.16137D+00  0.35890D+01  0.16621D+00  0.00000D+00
 0.10000D+01  0.00000D+00  0.10000D+01  0.15890D+01  0.00000D+00  0.11136D+02
...
```

For each repetition:

1. line: 1) number of repetition; 2) convergencec; 3) admissability;
   4)df ; 5) minimum fit chisquare; 6) significance of mfc;
   7) normal theory chisquare; 8) fit of ntc; 9) Satorra-Bentler scaled chisquare
2. line: 1) significance of S-Bsc; 2) chi corrected for nonormality; 3) sign. of ccnnM
   4) ncp; 5+6) confidence limits for ncp;
3. line: 1) fit function; 2) pdf; 3+4) confidence limits for pdf; 5) RMSEA 6) u. conf for RMSEA
4. line: 1) o conf for RMSEA; 2) sign. for test of close fit; 3) ECVI; 4+5) conf for ECVI ...

...

8. line: 1) RFI; 2) critical N

STEP 5:
Analyze the results using PRELIS:

```
Analyzing the parameter estimates in SIMULAT.PAR
DA NI=13
LA
'LX(2,1)' 'LX(3,1)' 'LX(3,2)' 'LX(5,2)'
'PH(1,1)' 'PH(2,1)' 'PH(2,2)'
'TD(1,1)' 'TD(2,2)' 'TD(3,3)' 'TD(4,4)' 'TD(5,1)' 'TD(5,5)'
RA=SIMULAT.PAR FO
(/(6D13.6))
CO 'LX(2,1)' - 'TD(5,5)'
OU MA=KM XM
```

Results:

```
Total Sample Size =    100
Univariate Summary Statistics for Continuous Variables
```

Variable	Mean	St. Dev.	T-Value	Skewness	Kurtosis	Minimum	Freq.	Maximum	Freq.
LX(2,1)	0.986	0.074	133.413	-0.099	-0.533	0.810	1	1.148	1
LX(3,1)	0.481	0.174	27.623	0.681	1.784	0.076	1	1.185	1
LX(3,2)	-0.217	0.179	-12.130	0.304	1.313	-0.781	1	0.388	1
LX(5,2)	0.809	0.087	92.664	0.405	0.196	0.623	1	1.052	1
PH(1,1)	1.034	0.132	78.096	0.223	-0.390	0.730	1	1.387	1
PH(2,1)	-0.806	0.098	-82.041	0.009	-0.067	-1.069	1	-0.537	1
PH(2,2)	0.991	0.152	65.244	-0.332	0.322	0.520	1	1.315	1

Variable	Mean	St. Dev.	T-Value	Skewness	Kurtosis	Minimum	Freq.	Maximum	Freq.
TD(1,1)	0.474	0.089	53.292	0.132	-0.722	0.268	1	0.678	1
TD(2,2)	0.515	0.088	58.891	-0.355	0.221	0.262	1	0.727	1
TD(3,3)	0.977	0.100	97.508	-0.161	-0.022	0.721	1	1.243	1
TD(4,4)	0.499	0.102	49.145	0.090	0.109	0.227	1	0.762	1
TD(5,1)	0.011	0.055	1.979	0.050	0.403	-0.152	1	0.151	1

Test of Univariate Normality for Continuous Variables

	Skewness		Kurtosis		Skewness and Kurtosis	
Variable	Z-Score	P-Value	Z-Score	P-Value	Chi-Square	P-Value
LX(2,1)	-0.409	0.682	-1.170	0.242	1.536	0.464
LX(3,1)	2.808	0.005	2.640	0.008	14.854	0.001
LX(3,2)	1.253	0.210	2.217	0.027	6.488	0.039
LX(5,2)	1.669	0.095	0.711	0.477	3.292	0.193
PH(1,1)	0.918	0.359	-0.689	0.491	1.317	0.518
PH(2,1)	0.039	0.969	0.170	0.865	0.030	0.985
PH(2,2)	-1.369	0.171	0.937	0.349	2.753	0.252
TD(1,1)	0.545	0.586	-1.945	0.052	4.080	0.130
TD(2,2)	-1.464	0.143	0.758	0.448	2.719	0.257
TD(3,3)	-0.663	0.507	0.270	0.787	0.513	0.774
TD(4,4)	0.370	0.712	0.545	0.586	0.433	0.805
TD(5,1)	0.208	0.835	1.071	0.284	1.189	0.552
TD(5,5)	-0.440	0.660	0.575	0.565	0.524	0.769

Correlation Matrix

	LX(2,1)	LX(3,1)	LX(3,2)	LX(5,2)	PH(1,1)	PH(2,1)
LX(2,1)	1.000					
LX(3,1)	-0.053	1.000				
LX(3,2)	-0.131	0.895	1.000			
LX(5,2)	-0.064	0.034	-0.044	1.000		
PH(1,1)	-0.438	-0.189	-0.106	0.035	1.000	
PH(2,1)	0.040	-0.016	-0.079	0.275	-0.657	1.000
PH(2,2)	0.133	0.047	0.063	-0.628	0.251	-0.737
TD(1,1)	0.595	0.023	-0.043	-0.061	-0.256	-0.010
TD(2,2)	-0.402	0.192	0.276	0.056	0.032	0.017
TD(3,3)	-0.038	-0.125	-0.055	0.059	0.285	-0.096
TD(4,4)	-0.066	0.079	0.010	0.593	0.048	0.053
TD(5,1)	-0.195	0.158	0.147	0.311	-0.001	-0.036
TD(5,5)	0.190	0.020	0.081	-0.555	-0.018	-0.216

	PH(2,2)	TD(1,1)	TD(2,2)	TD(3,3)	TD(4,4)	TD(5,1)
PH(2,2)	1.000					
TD(1,1)	0.115	1.000				
TD(2,2)	-0.129	-0.545	1.000			
TD(3,3)	-0.060	0.017	-0.041	1.000		
TD(4,4)	-0.455	-0.091	0.115	0.056	1.000	
TD(5,1)	-0.232	-0.298	0.224	0.161	0.470	1.000
TD(5,5)	0.345	0.079	-0.098	-0.147	-0.413	-0.372

	TD(5,5)
TD(5,5)	1.000

Bootstrapping

A method related to simulation is bootstrapping. If one is interested in the properties of an consistent estimator with unknown sample distribution from a random sample bootstrap samples can be drawn by select cases randomly with replacement from the empirical sample.

If q is a vector of parameters, $\hat{\theta}$ is a consistent estimation based on an empirical sample, $\hat{\theta}_i$ is the estimation in the i-th bootstrap sample and n_b is the number of bootstrap samples, one might approximate the asymptotic variance-covariance matrix by

$$\hat{\Sigma}_{\hat{\theta}} = \frac{1}{n_b} \sum_{i=1}^{n_b} (\hat{\theta}_i - \hat{\theta}) \times (\hat{\theta}_i - \hat{\theta})'$$

The realization using PRELIS and LISREL is similar to Monte-Carlo simulation:
First bootstrap-samples are generated by random sampling with replacement cases from given raw data,
then parameters are estimated in each bootstrap-samples,
and lastly means and standard deviations are computed across samples.

An Example:

```
Generating raw data
DA NO=250
new v1=NRAND
new v2=NRAND
new v3=NRAND
new v4=NRAND
new v5=NRAND
new x1= 1.224745*v1
new x2= 0.816497*v1-0.912871*v2
new x3= 0.538888*v1-0.240998*v2-1.049533*v3
new x4=-0.653198*v1+0.292119*v2+0.169218*v3+0.979472*v4
new x5=-0.521742*v1+0.234425*v2+0.135626*v3+0.375477*v4+0.808365*v5
sd v1-v5
CO ALL
OU MA=CM cm=boot.cm ac=boot.acc RA=bootstr.dat XM IX=123456789 XO
```

Generated sample covariance matrix:

	x1	x2	x3	x4	x5
x1	1.620				
x2	1.115	1.746			
x3	0.775	0.811	1.451		
x4	-0.875	-0.971	-0.657	1.605	
x5	-0.633	-0.685	-0.513	0.803	1.097

Step 1: Generating 100 bootstrap-samples with 100% sample size

```
Generating bootstrap samples
DA NI=5
RA=bootstr.dat
CO all
OU MA=CM BS=100 SF=100 BM=bootstr.bm XM IX=123456789 XO
```

Average of the 100 estimated covariance matrices

```
 Total Sample Size =    250
Bootstrap Covariance Matrix
               VAR 1      VAR 2      VAR 3      VAR 4      VAR 5
            --------   --------   --------   --------   --------
    VAR 1      1.541
    VAR 2      1.000      1.595
    VAR 3      0.708      0.695      1.286
    VAR 4     -0.870     -0.910     -0.632      1.491
    VAR 5     -0.524     -0.660     -0.412      0.734      0.996
```

Sample covariance matrix:

```
                  x1         x2         x3         x4         x5
            --------   --------   --------   --------   --------
       x1      1.620
       x2      1.115      1.746
       x3      0.775      0.811      1.451
       x4     -0.875     -0.971     -0.657      1.605
       x5     -0.633     -0.685     -0.513      0.803      1.097
```

STEP 2:Estimate the parameters in each of the generated samples:

```
Analyze bootstrap covariance matrices
DA NI=5 NO=250 MA=CM RP=100
CM=bootstr.BM
LA
X1 X2 X3 X4 X5
MO NX=5 NK=2 LX=FU,FI PH=SY,FR TD=SY
LK
Xi1 Xi2
VA 1.0  LX(1,1) LX(4,2)
FR LX(2,1) LX(3,1) LX(3,2) LX(5,2) TD(5,1)
OU  ND=3 XO=1 PV=bootstr.par SV=bootstr.se GF=bootstr.gf xi
```

xi on the ou-command reduces output of gf-file to chisquare statistics

STEP 3: Analyze the parameter estimates in BOOTSTR.PAR

```
Analyze the parameter estimates in BOOTSTR.PAR
DA NI=13
LA
'LX(2,1)' 'LX(3,1)' 'LX(3,2)' 'LX(5,2)'
'PH(1,1)''PH(2,1)' 'PH(2,2)'
'TD(1,1)' 'TD(2,2)' 'TD(3,3)' 'TD(4,4)' 'TD(5,1)' 'TD(5,5)'
RA=BOOTSTR.PAR FO
(/(6D13.6))
CO 'LX(2,1)' - 'TD(5,5)'
OU MA=KM XM
```

Results:

Total Sample Size = 100

Univariate Summary Statistics for Continuous Variables

Variable	Mean	St. Dev.	T-Value	Skewness	Kurtosis	Minimum	Freq.	Maximum	Freq.
LX(2,1)	1.082	0.111	97.090	0.844	1.550	0.857	1	1.502	1
LX(3,1)	0.703	0.201	34.882	1.019	2.487	0.311	1	1.561	1
LX(3,2)	-0.050	0.199	-2.503	1.379	5.209	-0.405	1	0.949	1
LX(5,2)	0.728	0.084	87.149	0.168	0.145	0.521	1	0.962	1
PH(1,1)	1.023	0.149	68.779	0.098	-0.206	0.704	1	1.379	1
PH(2,1)	-0.876	0.110	-79.932	-0.249	-0.718	-1.134	1	-0.681	1
PH(2,2)	1.106	0.151	73.430	0.365	-0.195	0.822	1	1.503	1
TD(1,1)	0.589	0.096	61.482	0.324	0.179	0.321	1	0.841	1
TD(2,2)	0.538	0.102	52.701	0.391	1.336	0.244	1	0.879	1
TD(3,3)	0.857	0.084	101.937	0.215	-0.006	0.667	1	1.123	1
TD(4,4)	0.499	0.113	44.234	-0.251	-0.318	0.204	1	0.719	1
TD(5,1)	0.009	0.061	1.420	-0.243	0.149	-0.179	1	0.140	1
TD(5,5)	0.500	0.064	78.576	0.085	0.209	0.338	1	0.685	1

	Skewness		Kurtosis		Skewness and Kurtosis	
Variable	Z-Score	P-Value	Z-Score	P-Value	Chi-Square	P-Value
LX(2,1)	3.236	0.001	2.344	0.019	15.970	0.000
LX(3,1)	3.762	0.000	3.059	0.002	23.511	0.000
LX(3,2)	4.704	0.000	4.296	0.000	40.588	0.000
LX(5,2)	0.716	0.474	0.489	0.625	0.751	0.687
PH(1,1)	0.417	0.676	-0.310	0.757	0.270	0.874
PH(2,1)	-1.052	0.293	-2.112	0.035	5.568	0.062
PH(2,2)	1.523	0.128	-0.282	0.778	2.399	0.301
TD(1,1)	1.361	0.173	0.554	0.579	2.161	0.339
TD(2,2)	1.626	0.104	2.141	0.032	7.229	0.027
TD(3,3)	0.913	0.361	0.173	0.863	0.864	0.649
TD(4,4)	-1.061	0.289	-0.621	0.535	1.510	0.470
TD(5,1)	-1.030	0.303	0.497	0.619	1.309	0.520
TD(5,5)	0.363	0.716	0.612	0.541	0.506	0.776

Results: Covariance Matrix

	LX(2,1)	LX(3,1)	LX(3,2)	LX(5,2)	PH(1,1)	PH(2,1)
LX(2,1)	0.012					
LX(3,1)	0.002	0.041				
LX(3,2)	-0.002	0.037	0.039			
LX(5,2)	-0.001	0.004	0.003	0.007		
PH(1,1)	-0.010	-0.009	-0.002	-0.003	0.022	
PH(2,1)	0.005	0.003	-0.001	0.004	-0.013	0.012
PH(2,2)	0.001	-0.003	-0.001	-0.008	0.007	-0.011
TD(1,1)	0.006	0.001	-0.001	-0.001	-0.005	0.003
TD(2,2)	-0.007	0.003	0.004	0.001	0.004	-0.003
TD(3,3)	0.000	-0.006	-0.005	-0.001	-0.001	0.001
TD(4,4)	-0.003	0.002	0.002	0.005	0.001	0.001
TD(5,1)	-0.003	0.002	0.003	0.002	0.002	-0.001
TD(5,5)	0.001	0.000	-0.001	-0.003	0.001	-0.002

	PH(2,2)	TD(1,1)	TD(2,2)	TD(3,3)	TD(4,4)	TD(5,1)
PH(2,2)	0.023					
TD(1,1)	0.000	0.009				
TD(2,2)	-0.001	-0.004	0.010			
TD(3,3)	0.001	0.001	0.000	0.007		
TD(4,4)	-0.009	-0.003	0.002	-0.001	0.013	
TD(5,1)	-0.002	-0.003	0.002	-0.001	0.002	0.004
TD(5,5)	0.005	0.001	-0.002	0.001	-0.004	-0.001

	TD(5,5)
TD(5,5)	0.004

Estimation of Asymptotic Covariance Matrix Using Bootstrapping

The bootstrap method can be used to estimate an acc-matrix :

```
Using bootstrap samples for computation of acc
DA NI=5
RA=bootstr.dat
CO all
OU MA=CM BS=100 SF=100 ac=bootstr.acc XM IX=123456789
```

Estimation of the model using LISREL

```
Robust estimation using bootstrapped acc
SY=bootstr5.dsf
MO NX=5 NK=2 LX=FU,FI PH=SY,FR TD=SY
LK
Xi1 Xi2
VA 1.0  LX(1,1) LX(4,2)
FR LX(2,1) LX(3,1) LX(3,2) LX(5,2) TD(5,1)
OU  ND=3
```

Results

Bootstrap-Sample:

Covariance Matrix

	VAR 1	VAR 2	VAR 3	VAR 4	VAR 5
VAR 1	1.619				
VAR 2	1.230	1.894			
VAR 3	0.769	0.727	1.270		
VAR 4	-1.015	-1.101	-0.671	1.863	
VAR 5	-0.664	-0.812	-0.497	0.945	1.136

Empirical Sample

Covariance Matrix

	x1	x2	x3	x4	x5
x1	1.620				
x2	1.115	1.746			
x3	0.775	0.811	1.451		
x4	-0.875	-0.971	-0.657	1.605	
x5	-0.633	-0.685	-0.513	0.803	1.097

Results

Bootstrap-Sample:

```
LISREL Estimates
(Robust Maximum Likelihood)
LAMBDA-X
                 Xi1          Xi2
              --------     --------
VAR 1         1.000          - -

VAR 2         1.031          - -
             (0.093)
              11.048

VAR 3         0.558        -0.074
             (0.172)       (0.167)
              3.237        -0.442

VAR 4          - -          1.000

VAR 5          - -          0.752
                           (0.077)
                            9.739
```

Empirical Sample:

```
LISREL Estimates
(Robust Maximum Likelihood)
 LAMBDA-X
                 Xi1          Xi2
              --------     --------
 x1           1.000          - -

 x2           1.078          - -
             (0.096)
              11.261

 x3           0.686        -0.061
             (0.188)       (0.181)
              3.655        -0.335

 x4            - -          1.000

 x5            - -          0.727
                           (0.082)
                            8.826
```

Results

Bootstrap-Sample:

PHI

	Xi1	Xi2
Xi1	1.193	
	(0.156)	
	7.639	
Xi2	-1.037	1.250
	(0.111)	(0.148)
	-9.363	8.444

Empirical Sample:

PHI

	Xi1	Xi2
Xi1	1.034	
	(0.158)	
	6.526	
Xi2	-0.885	1.105
	(0.112)	(0.145)
	-7.893	7.620

Results

Bootstrap-Sample:

THETA-DELTA

	VAR 1	VAR 2	VAR 3	VAR 4	VAR 5
VAR 1	0.426 (0.102) 4.162				
VAR 2	- -	0.626 (0.097) 6.430			
VAR 3	- -	- -	0.806 (0.079) 10.241		
VAR 4	- -	- -	- -	0.613 (0.111) 5.543	
VAR 5	0.110 (0.062) 1.781	- -	- -	- -	0.427 (0.068) 6.266

Results

Empirical Sample:

THETA-DELTA

	x1	x2	x3	x4	x5
x1	0.586 (0.090) 6.485				
x2	- -	0.544 (0.087) 6.270			
x3	- -	- -	0.888 (0.088) 10.119		
x4	- -	- -	- -	0.500 (0.097) 5.145	
x5	0.007 (0.053) 0.129	- -	- -	- -	0.513 (0.063) 8.086

Results

Bootstrap-Sample:

```
Goodness of Fit Statistics
Degrees of Freedom = 2
Minimum Fit Function Chi-Square = 1.983 (P = 0.371)
Normal Theory Weighted Least Squares Chi-Square = 1.975 (P = 0.373)
Satorra-Bentler Scaled Chi-Square = 1.587 (P = 0.452)
Chi-Square Corrected for Non-Normality = 1.605 (P = 0.448)
Estimated Non-centrality Parameter (NCP) = 0.0
90 Percent Confidence Interval for NCP = (0.0 ; 6.847)
Root Mean Square Error of Approximation (RMSEA) = 0.0
90 Percent Confidence Interval for RMSEA = (0.0 ; 0.117)
P-Value for Test of Close Fit (RMSEA < 0.05) = 0.638
```

Empirical Sample

```
Goodness of Fit Statistics
Degrees of Freedom = 2
Minimum Fit Function Chi-Square = 0.921 (P = 0.631)
Normal Theory Weighted Least Squares Chi-Square = 0.932 (P = 0.627)
Satorra-Bentler Scaled Chi-Square = 0.937 (P = 0.626)
Chi-Square Corrected for Non-Normality = 0.808 (P = 0.668)
Estimated Non-centrality Parameter (NCP) = 0.0
90 Percent Confidence Interval for NCP = (0.0 ; 5.035)
Root Mean Square Error of Approximation (RMSEA) = 0.0
90 Percent Confidence Interval for RMSEA = (0.0 ; 0.101)
P-Value for Test of Close Fit (RMSEA < 0.05) = 0.771
```

Exercises

1) Parametric Bootstrapping
From the efficacy-model based on ML-estimation with listwise deletion generate 1000 new samples of same sample size and compare the results of this simulation with the estimated standard errors and the chi-square goodness-of-fit statistic.
This method Bollen has called parametric bootstrapping.

2) Non-Parametric Bootstrapping
Use the ALLBUS raw-data and generate from this 1000 bootstrap-samples with
a) sample fraction =100%
b) sample fraction = 50%.
Compare again standard errors and chisquare of the resulting LISREL model.