Considering the Income-Illness Relation
Focussing on the Lower End of Conditional Health Distributions using Structured Additive Distributional Regression

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Abstract

In this paper we reconsider the relationship between income on health, taking a distributional perspective rather than one centered on conditional expectation. We find that the impact of income on health is is larger than generally estimated because aspects of the conditional health distribution that go beyond the expectation imply worse outcomes for those with lower incomes. For example, we find that the risk of very bad health is roughly halved by doubling the net equivalent income from 15,000€ to 30,000€, which is more than tenfold of the magnitude of change found when considering expected health measures. This paper therefore argues for the importance of a distributional perspective on health outcomes, which contemplates stochastic variation among observably equivalent individuals.

JEL-Classification:

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1 Introduction

Scores of papers assess the relationship between income and health status in a multivariate framework. Both in epidemiology and health economics, the vast majority of these employ standard regression methods (linear and generalized linear models) to assess the effect of variations of income and other covariates on the expectation of health status. While adequate in many scenarios,

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We thank...

this focus on expected health outcomes may neglect potentially important variation in health that is associated with differences in income. In this paper, we build on recent research that attempts to overcome the limitations of standard regression approaches and offer a more comprehensive analysis of health outcomes (Duclos and Échevin, 2011; Makdissi and Yazbeck, 2014). A transition from point estimation to distributional estimation has been made feasible by the evolution of computation capacity in the past decades, which now allows for rapid computation of the algorithms required for complex distributional regression models. We apply the recently developed technique of structured additive distributional regression (SADR) (Klein et al., 2015) to estimate the relationship between self-reported health status (in both a standard ordered 5-response format and in the more granular SF-12) and income, conditional on other standard covariates (e.g. age, education, etc.).

The association between income and health is one of the most robustly documented findings in the literatures on population health and health economics (Marmot, 2002; Kawachi et al., 2010). Income has been found to be strongly associated with measures of health across a variety of populations, even above a threshold of material deprivation (Backlund et al., 1996; Ettner, 1996; McDonough et al., 1997; Ecob and Davey Smith, 1999; Case, 2001), and recent studies exploiting exogenous variation in income have established causal effects of income on health(Kuehnle, 2014; Case, 2001; Frijters et al., 2005; Lindahl, 2005).

Although debate on the causal mechanisms linking income to health is not fully settled, the magnitude of the association between health and income is of critical concern for contemporary political decision making, particularly in areas such as the minimum wage, social minimum, or tax treatment of low earnings. In the literature on health economics, epidemiology and public health, estimates of the relationship between income and health have tended to take one of three forms: bivariate concentration indices summarizing the relationship between income and health in a population (Wagstaff and van Doorslaer, 1994; Lynch and Kaplan, 1997; Gravelle, 1998; Ecob and Davey Smith, 1999; Humphries and van Doorslaer, 2000; Gravelle, 2003; Lindahl, 2005; Wagstaff, 2005, 2011); estimates of the effect of income and other covariates on mean health status (Rogot et al., 1992; Ettner, 1996; Case, 2001; Contoyannis et al., 2004), and likelihood ratios that express the conditional probability of being in a particular health state given a particular level of income and other covariates (Benzeval et al., 2000; Frijters et al., 2005). The first two rely on health measures that are plausibly interpreted as continuous, while the latter technique is often used when the health outcome in question is measured using discrete (often binary) categories. Each of these approaches to measuring the relationship between income and health is useful, but, particularly when applied to the most widely used survey measures of health status, also has limitations.

Concentration indices are the "workhorse [method] in most health economic studies" (Fleurbaey and Schokkaert, 2009, p.73) for quantifying the distribution of health in a population. Their succinct form and resemblance to the Gini coefficient provide an intuitive scalar measure that is well suited for portraying the magnitude of health inequalities related to socio-economic characteristics (Wagstaff and van Doorslaer, 1994; Kakwani et al., 1997). Yet the concentration index by its construction only allows for the relation of two variables or scales. Estimating the relationship between health and income while jointly conditioning on a set of other covariates thought also to be relevant for health is thus not possible using conventional concentration indices. While we reconsider concentration indices later on, we leave this methodology aside for the moment and concentrate on those analyses of the health-income nexus which employ regression methodology to estimate the income-health relation while controlling for the effects of other covariates.

A second workhorse method, particularly prominent in the epidemiological literature, is the analysis of expected health outcomes expressed as odds ratios. In its most prevalent form, a logit model is used to predict outcomes on a binary health measure conditioning on a set of variables. One of the advantages of this method is that risk of having a particular outcome is easily related to the conditional expectation derived from the model. Due to the simplicity and popularity of this method, it is common practice to reduce health variables of higher complexity to a binary form in order to facilitate the construction of odds ratios (Chamberlain, 1980; Benzeval et al., 2000; Frijters et al., 2005). This reduction is problematic if the health outcome of interest does not adhere to the implicit assumptions required by such a reduction, including indifference among health outcomes grouped together in either of the dichotomous response categories. For some health measures, including general health status, a dichotomous representation of healthy/unhealthy is clearly insufficient as important variations would be disregarded.

Possibly for this reason, the construction of more fine-grained scales has become widespread for the analysis of health for various contexts. The most popular approach is to construct (quasi-)continuous health measures. These can subsequently be analyzed using simple mean regression models, like OLS. While these classical regression techniques have the capacity to generate information about the relationship between quasi-continuous health outcomes and other covariates, in practice the reported results generally attend solely to the conditional expectation derived from these models. As is the case with logit modeling described above, this reduction is problematic as potentially important variations beyond the mean are disregarded. For example, looking only at the mean health outcome conditional on covariates ignores research on the utility associated with varying health statuses, some of which suggests that an equal-sized change in health status above or below the mean may in practice generate asymmetric changes in well-being (Finkelstein et al., 2009).

One problem shared by both regression approaches is thus that they use only limited information from the full distribution of health in the sample, either by dichotomizing the outcome from

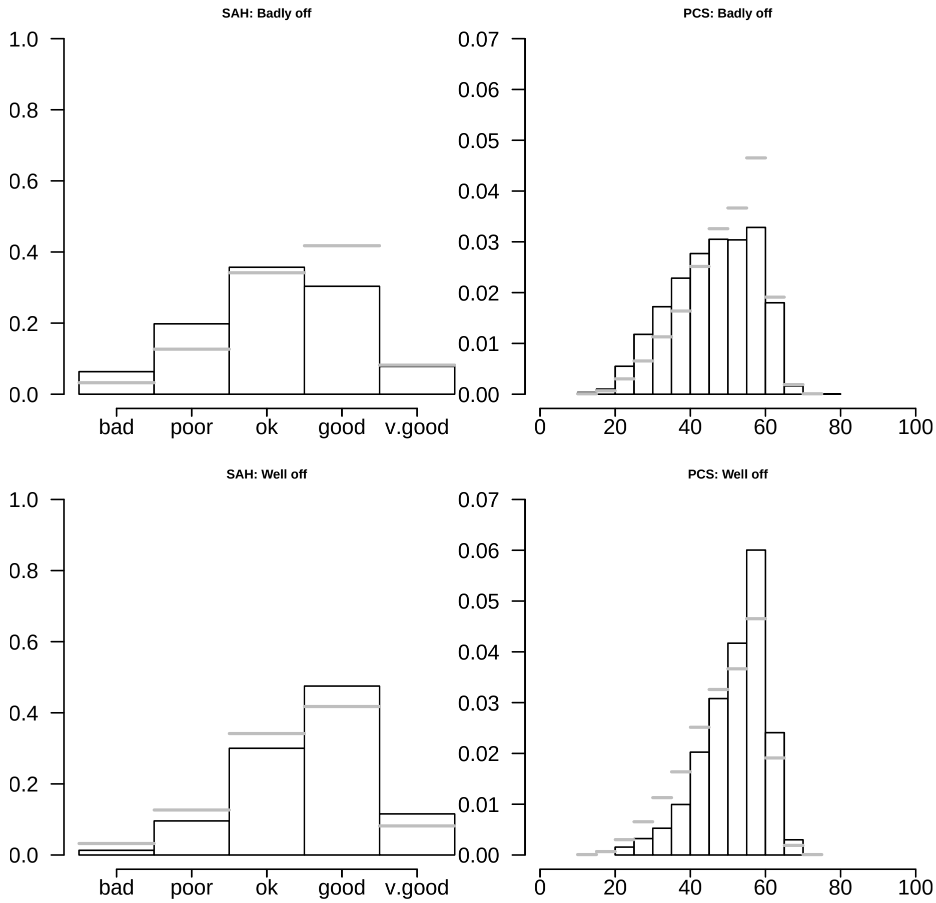

Figure 1: A contrast of coarsely conditioned health distributions. Top: Lowest 20% of net incomes. Bottom: Highest 20% of net incomes. Left: self-rated health (SRH). Right: Physical component score (PCS) of the SF-12. Grey lines indicate reference lines of middle group.

the outset or by considering only the conditional expectation of the outcome. Through such information reduction, these approaches focus attention on one particular aspect of the relationship between health and income (and/or other covariates) at the cost of ignoring other potentially important changes in the health distribution that may occur in connection with changes in income (and/or other covariates). While this narrowed perspective is adequate and indeed necessary in many scenarios, properly estimating the effect of income on health requires a broader approach.
A simple trisection of the health distribution between those with high, medium and low incomes reveals that the difference in the health outcomes by income go beyond differences in the mean. Figure 1 shows the distribution of two measures of generalized health – self-rated health (SRH) and a physical health score (PCS) – among those with high (top 20% w.r.t. net equivalent income) and low (bottom 20% w.r.t. net equivalent income) incomes. The variation in health outcomes is much more pronounced in the lower part of the income distribution, while those who are economically well off are able to practically eliminate the risk of very bad health. While an assessment based

on the distributions' means captures the general trend of the health-income relation, the reduction in information incurred by focussing on the mean leads us to underestimate the magnitude (and thus the real-world importance) of the relationship between income and health.

The technique we use in this paper, SADR, produces estimates of the conditional distribution of health that go beyond the expected health outcome. This approach leads to a starkly different assessment on the magnitude of the association between income and health, when controlling for a set of other covariates. For example, we find that for the "average Joe" and "average Jane," there is a difference in the risk of being severely ill of between 39% and 42% depending on whether the net equivalent household income is the median income of the poorer half of the population (15,000€) versus the median income of the richer half of the population (30,000€). The size of the this income effect is more than ten times the difference in the simple expectation of health status at these different income levels. The distributional approach also allows us to compare between discrete and continuous health measures, which turn out to have similar magnitudes for the income-health association. Thus distributional regression provides a shift in perspective which points to a significantly greater association between income and health than a perspective based on arithmetic averages alone would convey.

The structure of health care spending is generally such that far more resources are dedicated to improvements at the lower end of the health spectrum than to improvements at the higher end (Berk and Monheit, 2001). From this one may infer that at a societal level, the health-utility relationship is concave rather than linear. If the relationship is concave, it is obviously not linear – which is the only scenario in which a mean-based assessment would be warranted. Equally importantly, if the health-utility relationship is concave, analyses of the effects of income (or other covariates) on health ought to give more weight to the effects of covariate changes that affect the low end of the health distribution. For both multicategorical and continuous health measures, we thus propose the use of risk measures which focus explicitly on the lower end of the health distribution.

The remainder of this paper is structured as follows. The next section explains how SADR can be used to analyze conditional health distributions. In the subsequent section, we apply the approach to health data from the 2012 wave of the German socio-economic panel (SOEP), modeling the relationship between a discrete health score (self-rated health) and a quasi-continuous health score (SF-12) and net equivalent household income while controlling for a set of other variables like age, education, etc. We next illustrate the importance of taking a distributional perspective by highlighting several measures produced by SADR that put the emphasis on health impacts at the lower end of the spectrum. In the fourth and final section, we conclude.

2 Taking a Distributional Perspective

The conventional regression approaches discussed above fall into the category of generalized linear models, where the conditional expectation of a health outcome variable Y given a set of explanatory variables $x_1, \ldots, x_K$ is related to a regression predictor η via the response function h, i.e.

$$\mu = \mathbb{E}(Y \mid x_1, \ldots, x_K) = h(\eta).$$

The predictor in turn is usually modeled as a linear combination of the covariates[1] entailed in covariate vector $(x_1, \ldots, x_K)^T$, i.e.

$$\eta = \beta_0 + \sum_{k=1}^{K} \beta_k x_k.$$

For example, in case of binary outcomes differentiating only between healthy and non-healthy individuals, a logit or probit model is specified, in which the probability of an outcome $\pi = P(Y = 1 \mid x_1, \ldots, x_K) = \mathbb{E}(Y \mid x_1, \ldots, x_K)$ is related to the predictors via the cumulative distribution function of the logistic and the standard normal distribution, respectively.

The most important feature of generalized linear models for our purposes is that they focus exclusively on modeling the expectation of the response variable. Unlike in the case of binary responses, where the distribution of the health outcome is completely determined by the expectation (i.e. the success probability), when outcomes are more complex the expectation alone generally does not represent the complete distribution of the health outcomes well. We will analyze both multicategorical and continuous measures for health outcomes and in these cases the deviations from the expectation are typically at least as important as determinants of expected health. More importantly, these deviations may also be driven by covariates such that more general features of the health outcome distribution such as variance and skewness should also be modeled in terms of regression predictors.

A distributional perspective is needed to allow us to not just consider the conditional expectation of the health variable of interest, $\mathbb{E}(Y \mid x_1, \ldots, x_K)$, but also to relate the complete underlying conditional distribution, $\mathcal{D}(Y \mid x_1, \ldots, x_K)$ to the covariates. To achieve this goal, we could for rely on quantile regression as proposed by Koenker and Bassett (1978) to construct the distribution from the conditional quantiles. Alternatively, conditional transformation models as proposed by Hothorn et al. (2014) or the related distributional regression models proposed by Chernozhukov et al. (2013) could be used. We will rely on structured additive distributional regression (SADR) models as introduced in Klein et al. (2015), in which a parametric distribution type is assumed for

[1] More flexible alternatives have been developed in the context of generalized additive models (see Hastie and Tibshirani, 1990) or structured additive regression models (see Fahrmeir et al., 2004), but we will restrict ourselves to linear predictors in the following.

the conditional distribution $\mathcal{D}(Y \mid x_1, \ldots, x_K)$, but all parameters (not only the mean) are then related to regression predictors based on a suitably chosen response function. More specifically, we assume that the conditional distribution $\mathcal{D}(\theta_1(x_1, \ldots, x_K), \theta_2(x_1, \ldots, x_K), \ldots, \theta_L(x_1, \ldots, x_K))$ is characterised by a vector of L parameters $\theta_l(x_1, \ldots, x_K)$, $l = 1, \ldots, L$, and specify

$$g_l(\theta_l) = \eta^{\theta_l} \tag{1}$$

$$\eta^{\theta_l} = \beta_0^{\theta_l} + \sum_{k=1}^{K} \beta_k^{\theta_l} x_k. \tag{2}$$

Consequently, the vector of all regression coefficients $\boldsymbol{\beta}$ entails parameters not only for one predictor but for all L predictors required to specify the response distribution.

The main advantage of the parametric distributional approach, especially when Bayesian simulation is used to estimate the models, is that the models provide estimates and uncertainty measures not only for the regression effects themselves but for the complete conditional distribution. As a result, we are able to derive multiple health risk measures from these estimated distributions that are both easy to interpret and of significance for policy decisions, as they shift the focus towards the sick.

3 A Distributional Health Assessment for Germany

To illustrate the difference between a distributional perspective and conventional estimation methodologies, we consider a very simple application using health data from the German Socio-Economic Panel (SOEP, 2014).

3.1 The German Socio-Economic Panel

The German Socio-Economic Panel (SOEP) is a longitudinal household survey repeated annually since 1984 (Wagner et al., 2007). For this study we use only the cross-sectional data from the 2012 survey, which contains information on over 10,000 households (see SOEP, 2014; Rahmann and Schupp, 2013). The SOEP contains a rich array of sociodemographic information about individuals in these households, as well as several measures of health status. In this paper we consider both the standard five-response self-rated health item and the SF-12 physical health scale, as representative ordinal and (quasi-)continuous health measures, respectively. Thus we show that our proposed perspective is feasible for both discrete and continuous variables, both of which are frequently used in the literature. Indeed, as we will show, our proposed perspective which focusses on the poor yields similar outcomes irrespective of whether we use self-rated health or the SF-12 physical health scale. In the following, both health measures are related to a set of sociodemographic variables that are standard in the literature (see below). Using only those individuals for whom we have full information on these variables (see below), the 2012 SOEP yields 16,723 observations: 7,820 males and 8,903 females.

3.1.1 Self-rated health

In social epidemiological research, the most commonly used indicator of health status is generalized self-rated health (SRH), captured in a single item with a Likert response scale: How would you describe your current health?: Very good, good, satisfactory, poor or bad. Single-item SRH measures have been found in multiple populations to be reliable and responsive to changes in health status, and to predict health expenditure and outcomes (Idler and Benyamini, 1997; DeSalvo et al., 2006). Because well-being is intimately tied to one's sense of identity, single-item measures can tap respondents' ability to identify whether or not they are healthy quickly and holistically, and drawing on information that may not (yet) be available to their physicians or to researchers as diagnoses of specific conditions (Benyamini, 2011). DeSalvo et al. (2005, 2009) compare a standard single-item SRH measure to more comprehensive batteries and find that despite its brevity and simplicity, the single-item SRH is equally useful for predicting mortality, health care utilization, and health expenditures.

3.1.2 The SF12

Every two years since 2002, the SOEP has included a battery of health-related questions, the "SF-12v2$^{\text{TM}}$ Health Survey" (SF-12 Wagner et al., 2007). The SF-12 is a 12-item subset of Quality Metric's SF-36v2$^{\text{TM}}$, which is used widely in the recent literature (e.g. Marcus, 2013; LaMontagne et al., 2014; Eibich and Ziebarth, 2014) and provides measures of self-rated health in eight domains. The SF-12 comprises 12 items that aim to capture "practical, reliable and valid information about functional health and well-being from the patient's point of view" (OPTUM, 2015). Principal component analysis is used to compute two superordinate scales on physical health (PCS) and mental health (MCS), designed to have a mean of 50 and a standard deviation of 10. See Andersen et al. (2007) for details on the computation.

The SF-12 is an alternative to the longer SF-36 and to single-item measures of general self-rated health (SRH). The SF-12 has been found to be reliable, internally consistent, and to have good convergent and discriminant validity (Gandek et al., 1998; Franks et al., 2003; Bohannon et al., 2004; Cunillera et al., 2010). Across a variety of health outcomes and countries and with different patient populations, the SF-12 predicts physical and mental health outcomes, health related quality of life, and medical expenditure (see Ware Jr. et al., 1996; Fleishman et al., 2006). In cross-sectional and longitudinal tests of validity, the SF-12 generally yielded larger standard errors than the SF-36 (?). Nevertheless, the SF-12 is a practical and widely accepted tool for measuring population health and fo predicting health outcomes and expenditure. It has also been found to map reliably onto the EQ-5D scale, making it useful for generating the preference weightings needed to construct QALYs and other similar measures (Brazier and Roberts, 2004; Lawrence and Fleishman, 2004; Gray et al., 2006). The SF-12 has been found in previous studies to be correlated with income in a general population, even after adjusting for relevant covariates (Burdine et al., 2000; Schnittker, 2004; König et al., 2010). In our analysis we use only the PCS subscale of the SF-12. Differential item functioning by education, age and sex has been observed for the MCS (Fleishman and Lawrence, 2003; Bourion-Bédès et al., 2015), and since the SOEP does not include the institutionalized population, the sample is likely to be non-representative of the population with very low MCS scores.

3.1.3 The Explanatory Variables

As our main explanatory variable of interest, we consider disposable income, measured as the annual net equivalized household income of an individual, using the OECD equivalence scale to adjust for household size and composition. Additionally, we consider a set of variables to control for important socio-demographic variables that are correlated with income. For simplicity, we base our specification on variables typically found in a Mincer-type wage equation, as is done for example by Lorgelly and Lindley (2008).

For the annual net equivalized household income, we use the log transformation (LOGINC), which has been shown to be a suitable parametrisation by Jones and Wildman (2005). In addition to income, we consider the respondent's age as a quadratic polynomial (AGE and AGESQ), to control for differences in health induced by the inevitable biologically-induced deterioration of a persons health over the life course (see Kiuila and Mieszkowsko, 2007).

To adjust for the well known relationship between education (or cultural capital in a broader sense) and health, we control for respondent's educational attainment measured using the ISCED97 education categories provided by the SOEP. Here, we use four education levels. The first level (EDU_1) includes all individuals who have only general elementary education or less (i.e. those whose ISCED is between 0 and 2). The second level (EDU_2) entails all persons with completed secondary education (i.e. ISCED level 3) while the third level (EDU_3) entails all with ISCED levels 4 and 5, i.e. vocational training with *Abitur* or higher vocational training. The highest level (EDU_4) entails all those with completed higher education (i.e. ISCED level 6).

Culturally induced health differences and the potential for a healthy migrant effect (see Bjornstrom and Kuhl, 2014), are controlled for by additionally including a variable measuring whether the respondent is a German national (GER).

We also account for the marital status of the person by considering four different statuses: The first status (MAR_1) is to be living in a partnership (response item *married, living together* and response item *registered partnership, living together*), the second status (MAR_2) is to be separated (response item *married, separated* and response item *divorced*), the third status (MAR_3) is to be single (response item *single*) and the fourth status (MAR_4) is to be widowed (response item *widowed*).

Lastly, we control for local inequality and the general prosperity of the area in which respondents live using a hierarchical regional effect for the federal state of residence of the individual (DISTRICT) as is done by Eibich and Ziebarth (2014).

For further information on the variables see Section A.1 in the appendix.

3.2 Model specification

3.2.1 Choice of the Response Distribution

As pointed out in Section 2, the we need to specify a suitable parametric distribution that is able to approximate the empirically observed conditional health distributions.

Self-rated health outcomes are measured on an ordinal five point scale, which means that their distribution can be characterized by four probability parameters. We use a sequence of logit models to differentiate between the five levels of the self-rated health score rather than to differentiate only between two amalgamations of the levels as is standard in the literature. We first regress the lowest response versus all higher health scores to differentiate low values of the score from all higher scores. In the second step, we consider only individuals that reached at least the second response level of the discrete health measure and contrast the second level it to all higher levels. Continuing this sequence for higher levels provides us with a set of sequential logit models that characterize the multinomial nature of the categorical health outcome while simultaneously acknowledging the ordinal structure in a simple and interpretable fashion.[2]

Scores on continuous health measures, such as the SF-12, generally deviate significantly from a symmetric distribution, such that regression specifications based on the normal distribution often

[2]Standard cumulative regression models for ordinal responses would be much more limited in their flexibility since they would restrict covariate effects to be the same for the transition between all different stages of the response.

do not provide sufficient flexibility. For the PCS, we find that the conditional health distributions generally feature a negative skewness and are thus converse to the more common symmetric or positively skewed distributions for which which most parametric formulations are tailored. To be able to employ well-established estimation routines for the standard parametric distributions, we follow Erreygers and van Ourti (2011) and use a linear transformation g_{PCS} of the health score

$$g_{PCS}(H) = H^* = \frac{(H_0 - H)}{H_{scale}}, \tag{3}$$

where H and H^* denote the untransformed and the transformed PCS health score respectively, while H_0 is a constant ensuring that H^* has a positive support if required. Lastly, H_{scale} is another constant rescaling the transformed health score. In the following, we will use $H_0 = 100$ and $H_{scale} = 10$ ensuring that our transformed health score is not only positive but also restricted to the interval $(0, 10)$ which enhances numerical stability. Subsequently, we estimate the conditional distributions of the transformed PCS using the well-known two parameter gamma distribution.[3] Once this conditional distribution is estimated, one can easily obtain the conditional distribution of the original PCS measure by simply applying the inverse transform, g_{PCS}^{-1}. Note that the gamma distribution is invariant under scaling such that we effectively model a shifted, reversed, scaled gamma distribution for the health scores.

For both the categorical self-rated health scores and the (quasi) continuous SF-12, we thus specify parametric conditional health distributions which require, respectively, four and two parameters to be estimated with respect to the covariates. With the two distribution types chosen, let us now turn to the specification of the predictors of the distributions' parameters.

3.2.2 Predictor specification

Let us now turn to the specification of predictors For the sake of simplicity, we will specify one generic predictor set-up which is applied to all parameters, i.e.

$$\begin{aligned}\eta_l = {} & \beta_0^{\theta_l} + \beta_1^{\theta_l}\mathrm{AGE} + \beta_2^{\theta_l}\mathrm{AGE}SQ + \beta_3^{\theta_l}\mathrm{LOGINC} + \beta_4^{\theta_l}\mathrm{GER} + \beta_5^{\theta_l}\mathrm{EDU}_2 + \beta_6^{\theta_l}\mathrm{EDU}_3 + \beta_7^{\theta_l}\mathrm{EDU}_4 \\ & + \beta_8^{\theta_l}\mathrm{MAR}_2 + \beta_9^{\theta_l}\mathrm{MAR}_3 + \beta_{10}^{\theta_l}\mathrm{MAR}_4 + \beta_{11}^{\theta_l}\mathrm{EAST} + \gamma_{\mathrm{DISTRICT}}^{\theta_l}\end{aligned} \tag{5}$$

[3] Using a representation of the gamma distribution where μ is the expectation parameter and s the shape parameter, we can write its density as:

$$p(y \mid \mu, s) = \left(\frac{s}{\mu}\right)^s \frac{y^{s-1}}{\mathrm{G}(s)} \exp\left(-\frac{s}{\mu}y\right), \tag{4}$$

where y denotes the transformed PCS outcome, which is H^* in our case and where G denotes the Gamma function.

where η_l is the predictor for for the lth parameter of the response distribution. The explanatory variables (defined as outlined in Section 3.1.3) are all included in a linear fashion, supplemented by two effects representing spatial variation in health outcomes. EAST is an effect-coded binary variable scored one if the federal state is in the east of Germany, thus capturing the structural differences between the former German Democratic Republic (GDR) and the Federal German Republic (FDR). The differences within the former GDR and FDR are captured by random effects, denoted by γ_{DISTRICT}. This regularizing approach is chosen over a plain use of fixed effects for all federal states in order to enhance estimation stability (see Klein et al., 2015).

In order to relate the predictors to their corresponding parameters, we specify appropriate response functions. For the categorical responses, these are simply given by logit response functions while the exponential response function is used to ensure positivity of the two parameters for the gamma distribution.

3.3 Parameter Estimates

The estimation is done in the software BayesX (Belitz et al., 2015) which employs Markov Chain Monte Carlo (MCMC) simulation techniques to estimate posterior distributions in a Bayesian framework. See Klein et al. (2015) for details on the estimation procedure. In the following set-up, we use non-informative flat priors for the linear effect. For the spatial effect, we use Gaussian random effects priors centered on zero with inverse gamma distributions (with hyperparameters $a = b = 0.001$) used as hyperpriors for their variance. To obtain the posterior distribution, we draw on one million MCMC realizations which are thinned out at a rate of 800 after a burn-in of 200,000 MCMC realizations. For the posterior distributions we thus obtain 1,000 MCMC realizations for each parameter.

Before we go on to discuss our main findings concerning the impact of income on the two health variables considered, we first portray the effects of all covariates on the predictors of the parameters required to yield the distribution. While some of the parameters are interpretable in their own right (for example μ for the gamma distribution), we focus on evaluating the resultant distribution rather than the single parameters' estimates.

Table 1 displays the estimates for the covariate effects on the predictors of the sequential logits for the self-rated health outcomes. Here, we display the medians of the posterior distributions with the 95% (symmetric) credible intervals denoted in the brackets. In order to conserve space, we do not display the estimates for the the random effect estimates for the individual federal states but show them separately in Table 5 in the appendix.

While the parameter $\tilde{\pi}_l$ can be interpreted individually, we will not analyze these effects in detail.

	$\eta^{\tilde{\pi}_1}$	$\eta^{\tilde{\pi}_2}$	$\eta^{\tilde{\pi}_3}$	$\eta^{\tilde{\pi}_4}$
males				
const.	1.499[1.415; 1.592]	2.842[2.786; 2.902]	2.896[2.832; 2.969]	-0.899[-1.036;-0.742]
AGE	0.084[0.083; 0.085]	0.038[0.037; 0.038]	0.025[0.025; 0.026]	0.068[0.066; 0.070]
AGE^2	-0.001[-0.001;-0.001]	0.000[0.000; 0.000]	0.000[0.000; 0.000]	-0.001[-0.001; 0.000]
LOGINC	-0.182[-0.187;-0.178]	-0.458[-0.461;-0.455]	-0.453[-0.458;-0.450]	-0.298[-0.307;-0.290]
GER	-0.207[-0.211;-0.203]	-0.003[-0.006;-0.001]	0.266[0.262; 0.269]	-0.124[-0.132;-0.117]
EDU_2	0.024[0.020; 0.028]	0.069[0.067; 0.071]	-0.011[-0.014;-0.008]	0.148[0.142; 0.154]
EDU_3	0.064[0.059; 0.070]	-0.133[-0.136;-0.129]	-0.063[-0.068;-0.058]	-0.172[-0.184;-0.161]
EDU_4	-0.439[-0.443;-0.434]	-0.246[-0.248;-0.242]	-0.153[-0.157;-0.148]	-0.117[-0.127;-0.107]
MAR_2	-0.103[-0.110;-0.097]	0.215[0.211; 0.218]	-0.030[-0.035;-0.026]	0.153[0.145; 0.161]
MAR_3	-0.140[-0.146;-0.134]	0.084[0.081; 0.088]	0.326[0.321; 0.331]	0.056[0.046; 0.066]
MAR_4	0.158[0.149; 0.167]	-0.201[-0.205;-0.196]	-0.093[-0.097;-0.088]	-0.152[-0.161;-0.143]
EAST	0.247[0.072; 0.423]	-0.002[-0.099; 0.095]	-0.031[-0.100; 0.036]	0.037[-0.139; 0.207]
females				
	$\eta^{\tilde{\pi}_1}$	$\eta^{\tilde{\pi}_2}$	$\eta^{\tilde{\pi}_3}$	$\eta^{\tilde{\pi}_4}$
const.	-1.787[-1.863;-1.692]	1.266[1.209; 1.330]	4.003[3.918; 4.089]	-1.052[-1.216;-0.890]
AGE	0.142[0.141; 0.143]	0.092[0.091; 0.093]	0.021[0.020; 0.022]	0.034[0.032; 0.035]
AGE^2	-0.001[-0.001;-0.001]	0.000[0.000; 0.000]	0.000[0.000; 0.000]	0.000[0.000; 0.000]
LOGINC	0.004[0.000; 0.008]	-0.461[-0.464;-0.458]	-0.593[-0.598;-0.589]	-0.160[-0.169;-0.152]
GER	-0.106[-0.110;-0.101]	-0.267[-0.270;-0.264]	0.009[0.004; 0.013]	-0.071[-0.080;-0.063]
EDU_2	0.082[0.079; 0.086]	0.040[0.038; 0.043]	0.091[0.088; 0.095]	0.312[0.305; 0.319]
EDU_3	-0.064[-0.070;-0.059]	0.027[0.024; 0.031]	-0.071[-0.076;-0.066]	-0.343[-0.354;-0.332]
EDU_4	-0.424[-0.428;-0.419]	-0.334[-0.337;-0.331]	-0.007[-0.012;-0.003]	-0.159[-0.169;-0.149]
MAR_2	-0.084[-0.093;-0.074]	0.167[0.163; 0.172]	0.006[0.001; 0.011]	0.248[0.238; 0.258]
MAR_3	-0.394[-0.403;-0.386]	0.209[0.205; 0.213]	0.051[0.046; 0.057]	-0.071[-0.082;-0.060]
MAR_4	0.442[0.424; 0.462]	-0.230[-0.236;-0.223]	0.016[0.009; 0.024]	-0.057[-0.071;-0.044]
EAST	0.092[-0.077; 0.264]	-0.024[-0.122; 0.072]	-0.009[-0.120; 0.100]	-0.089[-0.333; 0.148]

Table 1: Linear effects on $\eta^{\tilde{\pi}_1}$,$\eta^{\tilde{\pi}_2}$,$\eta^{\tilde{\pi}_3}$ and $\eta^{\tilde{\pi}_4}$ for PCS.

Here, we restrict ourselves to noting that the effects of various variables differ significantly across the range of parameters estimated, both for males and females. Regarding LOGINC in particular, the effects are significantly different at the 5% level for different parameters.

Table 2 shows the estimates for the predictors η^{μ} and η^{s} analogously to the table above. Again it may be noted that the effects are significantly different for males and females and that both for μ and for s, various covariates are significantly different from zero. For μ, which yields the conditional expectation, it should be noted that due to the linear transformation the effects are reversed, so that for example LOGINC has a negative impact on the predictor but thus a positive impact on the expected health, as one would expect. Concerning s, note that although a direct interpretation of the parameter is not feasible, one can observe that LOGINC as well as other variables have a significant impact which indicates complex changes across the covariate space that go beyond the changes in the conditional mean on which standard regression techniques focus.

	males		females	
	η^μ	η^s	η^μ	η^s
const.	1.637[1.562; 1.713]	3.382[2.709; 4.029]	1.777[1.700; 1.852]	3.186[2.554; 3.760]
AGE	0.007[0.006; 0.008]	-0.045[-0.057;-0.034]	0.005[0.004; 0.007]	-0.031[-0.043;-0.021]
AGE^2	0.000[0.000; 0.000]	0.000[0.000; 0.000]	0.000[0.000; 0.000]	0.000[0.000; 0.000]
LOGINC	-0.034[-0.041;-0.027]	0.152[0.091; 0.211]	-0.041[-0.048;-0.034]	0.123[0.066; 0.183]
GER	-0.010[-0.016;-0.004]	0.045[-0.017; 0.103]	0.007[0.001; 0.012]	0.042[-0.011; 0.099]
EDU_2	0.014[0.009; 0.019]	-0.077[-0.125;-0.032]	0.008[0.002; 0.013]	-0.079[-0.123;-0.036]
EDU_3	-0.006[-0.013; 0.001]	0.084[0.016; 0.147]	-0.013[-0.021;-0.005]	0.038[-0.029; 0.101]
EDU_4	-0.038[-0.044;-0.031]	0.062[0.000; 0.126]	-0.026[-0.033;-0.019]	0.017[-0.045; 0.080]
MAR_2	0.008[-0.002; 0.019]	0.023[-0.063; 0.105]	0.002[-0.006; 0.011]	0.000[-0.071; 0.063]
MAR_3	0.006[-0.004; 0.015]	-0.028[-0.107; 0.056]	-0.002[-0.011; 0.006]	-0.059[-0.128; 0.014]
MAR_4	-0.019[-0.036;-0.003]	-0.115[-0.246; 0.007]	-0.003[-0.013; 0.007]	-0.001[-0.081; 0.075]
EAST	0.010[-0.001; 0.021]	0.008[-0.074; 0.094]	0.009[-0.002; 0.019]	0.004[-0.044; 0.049]

Table 2: Linear effects on η^μ and η^s for PCS.

3.4 Considering the Distributional Changes

Since we employ non-linear link functions for our predictors, the impact of the variables varies across the covariate space. This is well known from the literature on generalized linear models (Nelder and Wedderburn, 1972). We thus employ effect displays as proposed by Fox (1987). This means that we consider the effect of varying income while the other covariates are fixed at a given value. Here we consider the effects for both males and females who can be considered the "average Joe/average Jane", i.e. who are 52 years of age, are married, live in North-Rhine Westphalia (the most populous state in Germany), have standard secondary education and have German nationality.[4] See Section A.4 in the appendix for other covariate combinations.

To visualize how the distribution of self-rated health changes with income, we display in Figure 2 the change in the probability of falling in one of the five health categories as one moves from the bottom to the top of the income distribution, as derived from the median results displayed in Table 1. We consider the income range from 5,000€ to 100,000€. The former constitutes the lower bound as only 1% of our estimates fall below this sample due to social security levels in Germany, while the latter is chosen as the upper bound as it roughly constitutes the threshold to the most well-off 1% of the population. In this range we thus cover the whole population bar the bottom and the top percent of the income distribution.

This visualization makes clear that the nature of the change in the health distribution across the income distribution is complex, and that dichotomizing the outcome, e.g. by subsuming the levels 1-2 (not healthy) and 3-5 (healthy), is likely to submerge important variation within the aggregated

[4]See Section A.1 in the appendix for the covariate distribution underlying this choice. For the continuous variable age we consider the arithmetic mean in our sample, while for the other categorical variables we consider the mode. Note also that it would be possible to consider average marginal effects rather than the marginal effects at the representative values. For the purposes of our paper, the marginal effects at the representative values were deemed more intuitive and are thus considered in the the following.

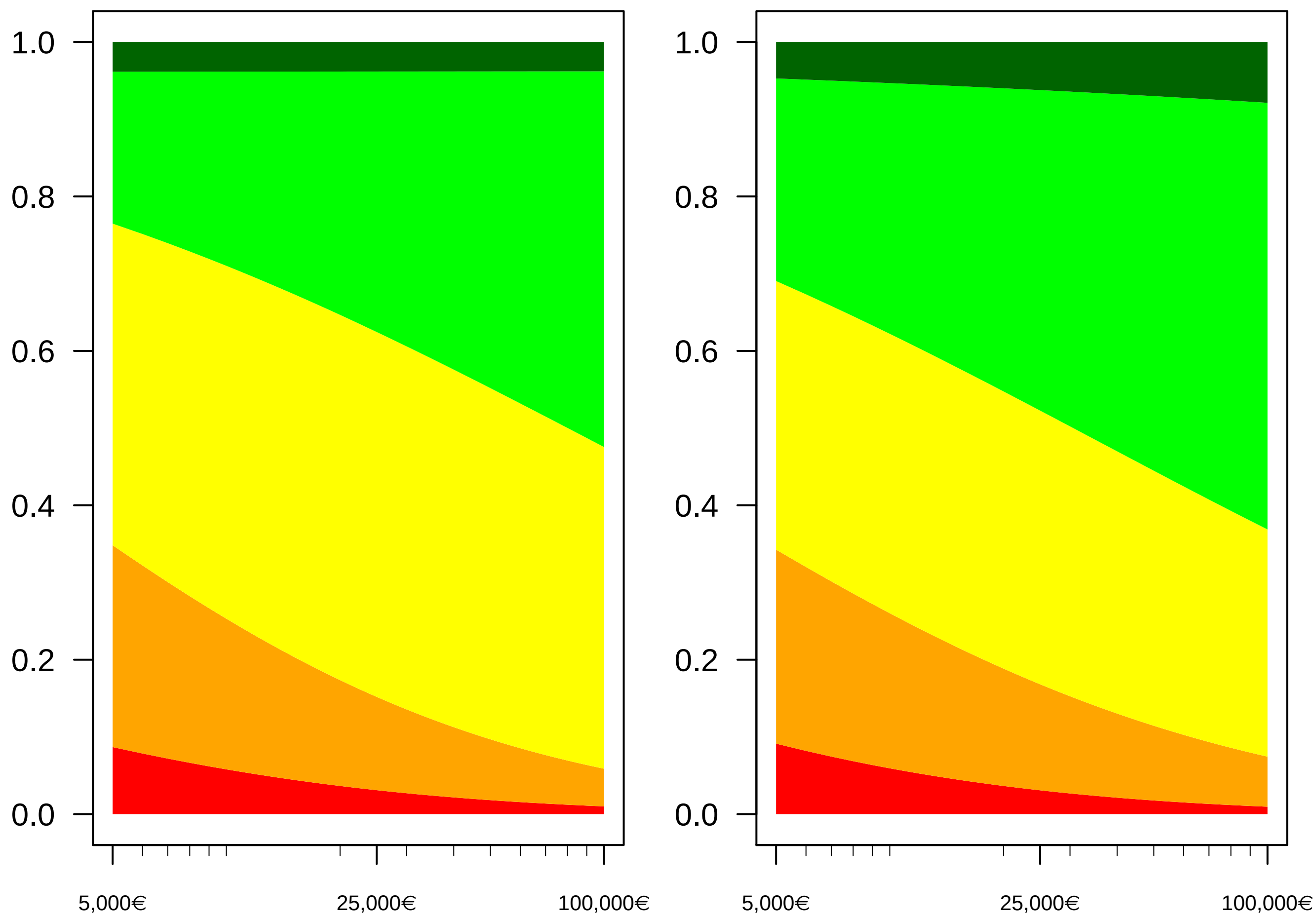

Figure 2: Income effect for self-rated health for men (left) and women (right). From red (poor health) to dark green (very good health).

categories.

In Figure 3, we zoom in on the difference in the conditional distributions of health status for men and women with a net equivalent income of 15,000€ (roughly corresponds to the 25th percentile, i.e. the median for the poorer half of the population) versus 30,000€(roughly corresponds to the 75th percentile, i.e. the median for the richer half of the population), with the other covariates fixed at the values to yield "average Joe" and "average Jane". The largest absolute differences occur near the center of the health distribution, i.e. for poor, fair, and good health. Despite the lower absolute levels, there are also noticeable changes at the bottom end of the health scale when moving from the lower to higher income level. Meanwhile, there is little change at the higher end of the distribution. This indicates that (more) money cannot buy (more) good health; but income does seem to contribute significantly to safeguarding against bad health outcomes – especially very bad ones, as we will see.

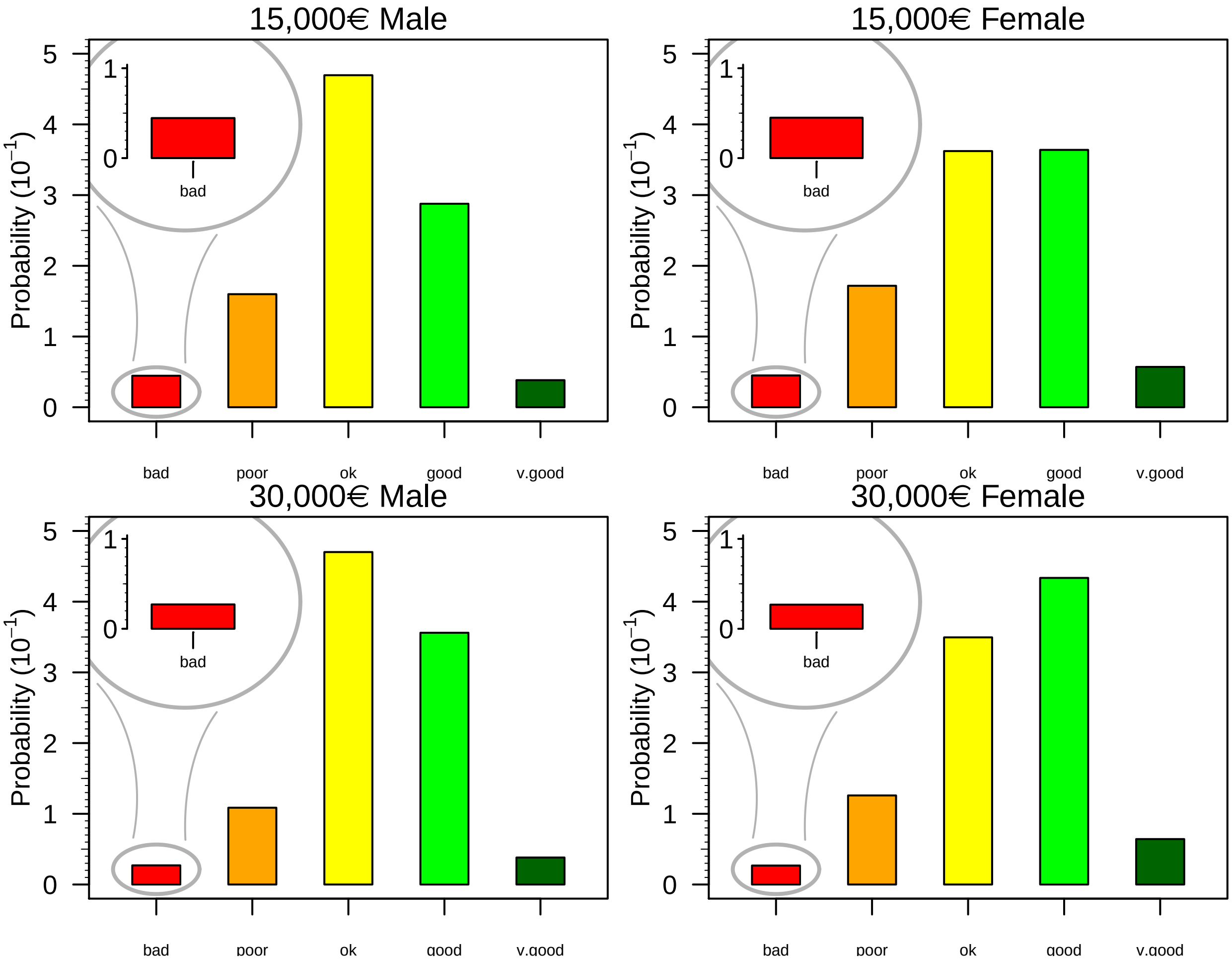

Figure 3: Conditional Health Distributions (SRH) for 15,000€ (top) and 30,000€ (bottom) for men (left) and women (right). With added focus on $\mathcal{R}_{M1}$.

Let us contemplate the risk of falling in one of the lowest response categories for health across the two distributions (for income of 15,000€versus 30,000€). We can define the following three health measures, which dichotomise the distribution in three different ways:

$$\begin{aligned}
\mathcal{R}_{M1} &= P(H^{\mathrm{M}} \leq \text{bad health}) \quad \text{with } H^{\mathrm{M}} \sim \mathcal{D}^{\mathrm{M}}_{\mathbf{x}}, \\
\mathcal{R}_{M2} &= P(H^{\mathrm{M}} \leq \text{poor health}) \quad \text{with } H^{\mathrm{M}} \sim \mathcal{D}^{\mathrm{M}}_{\mathbf{x}}, \\
\mathcal{R}_{M3} &= P(H^{\mathrm{M}} \leq \text{ok health}) \quad \text{with } H^{\mathrm{M}} \sim \mathcal{D}^{\mathrm{M}}_{\mathbf{x}},
\end{aligned}$$

where the health measures $\mathcal{R}_{M1}, \mathcal{R}_{M2}, \mathcal{R}_{M3}$ simply denote the risk of falling in one of the lowest response categories as given by the multinomial health distribution $\mathcal{D}^{\mathrm{M}}_{\mathbf{x}}$ which is dependent on the covariate combination under consideration, $\mathbf{x}$.

$\mathcal{R}_{M3}$ subsumes all health statuses below good into one category, thus representing the risk of "not feeling good about one's health". The probability of falling into one of these three lowest categories changes from 0.67 to 0.61 among men when moving from the conditional distribution for 15,000€ to that for 30,000€ – a change of 10%. For women, the probability falls from 0.58 to 0.50, a change of 13%. Although these differences are statistically significant, the magnitude is not substantively grave.

Secondly, we consider $\mathcal{R}_{M2}$, which by construction directs the attention towards those who are in poor or bad health (the bottom two health categories). This measure can therefore be seen as the risk of not only "not feeling good" but as "not even feeling ok". The change is of similar magnitude in absolute numbers, but much greater in relative terms. When income is doubled for men, the risk of low health status decreases by 34%, from 0.20 to 0.14, for men, while for women it falls 30%, from 0.22 to 0.15. The income-related change in risk of low health status is thus roughly 2-3 times as great when we aggregate the bottom two health categories as when we consider the bottom three categories together.

The third measure, $\mathcal{R}_{M1}$, is the most extreme measure which focusses on those who self-report a truly bad health. Thus it expresses the risk of positively "feeling bad about one's health". For this measure, the relative numbers are even more striking, with the probability of low health status decreasing by 39% and 40% for men and women respectively (from .04 to .03) as income doubles. The comparison of the three measures thus shows that the impact of household income on health seems to be much more drastic at the lower end of the self-rated health variable. Not surprisingly, this is also true when we consider the quasi-continuous PCS health score.

To characterize the relationship between income and the risk of low health using the SF12, we display six distributional measures in Figure 4. The blue line denotes males and the red line females, with the dashed lines denoting the 95% pointwise credible intervals.

The left-hand panels in 4 show the expectation (μ), the standard deviation(σ) and the skewness (γ_1) of the conditional distribution of the SF-12 across the full range of income. Note that we display these measures for the untransformed, original PCS variable, so that the effects are directly interpretable. The right-hand panels depict three measures of the risk of low health analogous to the ones used above. We portray the conditional probability that a person will fall below threshold values on the PCS scale representing the lower half (i.e. in the lowest 50%, denoted $T_{0.50}$), the lowest quintile (i.e. the lowest 20%, denoted $T_{0.20}$) and the lowest vingtile (i.e. the lowest 5%, denoted $T_{0.05}$) of the aggregate health distribution, depending on their income.[5] These measures can thus be seen as analogous variants of the risk measures $\mathcal{R}_{M1}$, $\mathcal{R}_{M2}$ and $\mathcal{R}_{M3}$ from above, indicating the risk of bad health. The measure $\mathcal{R}_{C0.50}$ thus yields the level of risk of belonging to the lower half of the health distribution, which can be seen as roughly equivalent to "not feeling good about one's health". Accordingly, $\mathcal{R}_{C0.20}$ yields the level of risk of belonging to the "sickest"

[5]These values are obviously not the only viable options but chosen on the grounds as to provide roughly analogous risk measures to the risk measures based on the self-rated health responses. More research is needed concerning the use of adequate scalar measures to assess this and other aspects of conditional health distributions.

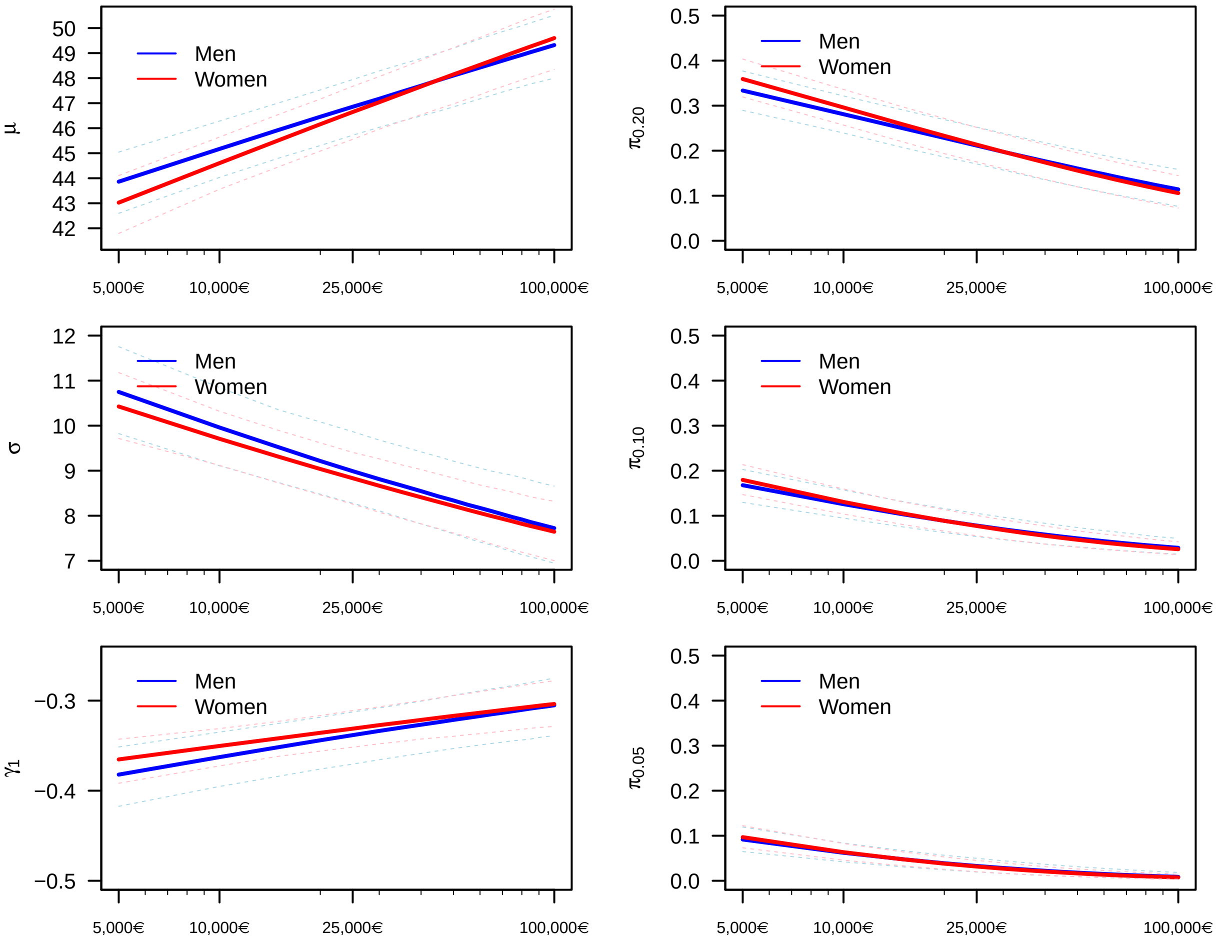

Figure 4: Left: Effect of income on mean, standard deviation and skewness of PCS. Right: Effect of income on risk of falling below lowest quintile, decile and vingtile of PCS.

20% of the population, which can be seen as roughly equivalent to people associating the health status as slightly sick, that is no longer "o.k.". Lastly, $\mathcal{R}_{C0.05}$ denotes the risk of falling into the lowest 5% of the health distribution, which would be associated with severe sickness and thus can roughly be seen as the equivalent to a person positively "feeling bad about one's health". More formally, the second set of risk measures can be defined as

$$\begin{aligned}
\mathcal{R}_{C0.05} &= P(H^{\mathrm{C}} \leq T_{0.05}) \quad \text{with } H^{\mathrm{C}} \sim \mathcal{D}^{\mathrm{C}}_{\mathbf{x}}, \\
\mathcal{R}_{C0.20} &= P(H^{\mathrm{C}} \leq T_{0.20}) \quad \text{with } H^{\mathrm{C}} \sim \mathcal{D}^{\mathrm{C}}_{\mathbf{x}}, \\
\mathcal{R}_{C0.50} &= P(H^{\mathrm{C}} \leq T_{0.50}) \quad \text{with } H^{\mathrm{C}} \sim \mathcal{D}^{\mathrm{C}}_{\mathbf{x}},
\end{aligned}$$

where the health variable H^{C} is now considered as continuous. The risk is thus given by the conditional distribution, $\mathcal{D}^{\mathrm{C}}_{\mathbf{x}}$, which the variable is thought to follow for an individual with characteristics $\mathbf{x}$, and the threshold value T_α, which we take to be a quintile from the aggregate distribution of

H^{C}.

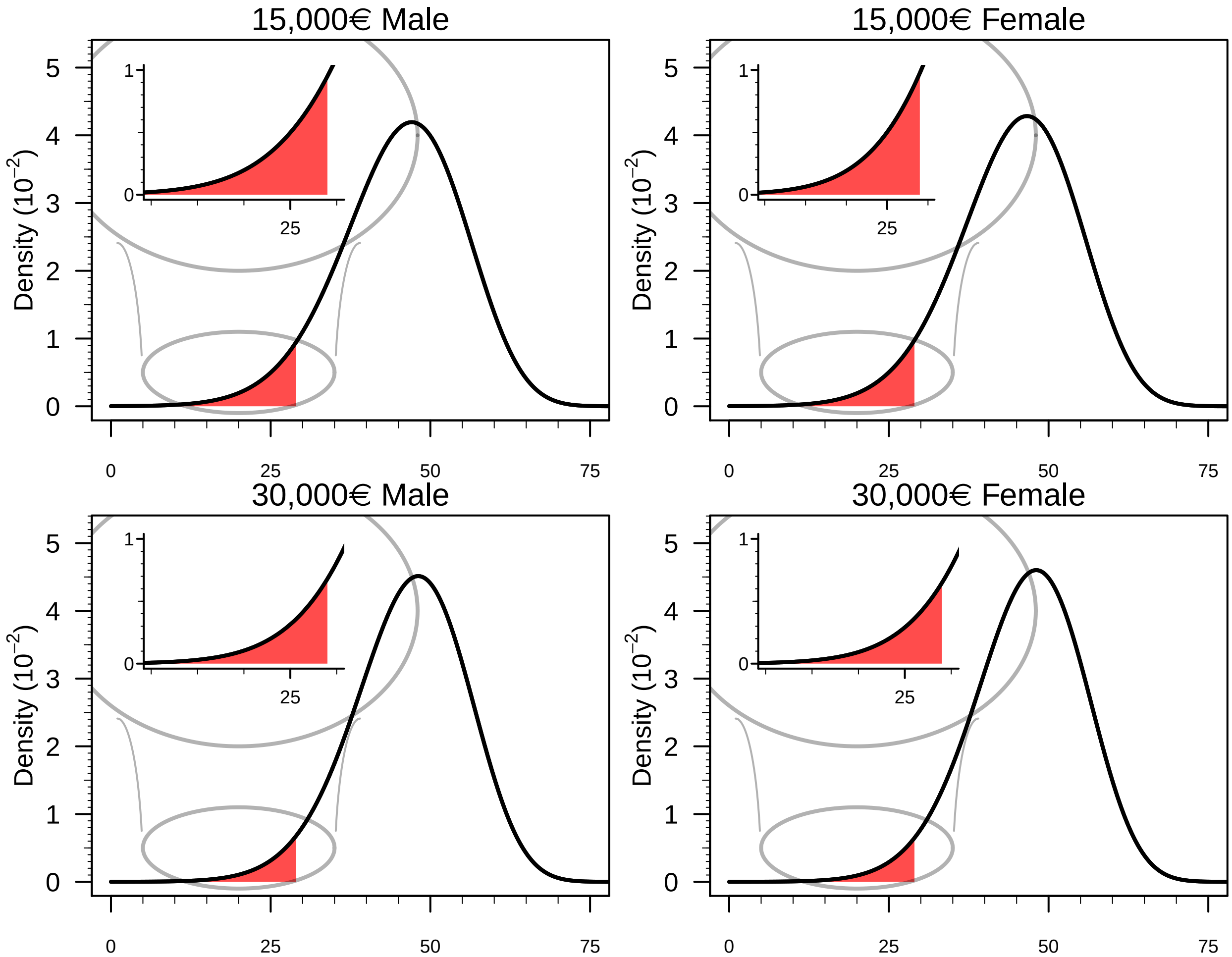

Figure 5: Conditional health distributions (PCS) for average Joe (left) and average Jane (right) for 15,000€ (top) and 30,000€ (bottom). With added focus on $\mathcal{R}_{C0.05}$.

The estimated full conditional distributions for the SF-12 for "average Joes" and "average Janes" are displayed in Figure 5. Again we focus on the contrast between 15,000€, representing the median income level of the poorer half of the sample population, and 30,000€, representing the median income level of the richer half of the sample population. While the displayed distributions appear rather similar at the first glance, a closer look at the different distributions' attributes reveals some substantial differences. For an annual net equivalent income of 15,000€ the average physical health value is 45.3 and 45.1 for men and women respectively. In contrast, for an income of 30,000€ , we obtain 46.7 and 46.6. Thus the average male described above with a net equivalent income of 30,000€ roughly has a 3% higher expected physical health score as an otherwise equivalent male with a net equivalent income of 15,000€. For a female the difference is also roughly 3%. This effect is well known and discussed extensively in the literature.

Next to the mean, the standard deviation also decreases from 9.5 to 8.8 and 9.3 to 8.7 for men and women respectively. This 7% decrease means that men and women with higher income face a lower risk to experience very low health outcomes for a given mean. Additionally, the distribution becomes slightly more right skewed, with the skewness increasing from -0.4 to -0.3 for both men and women. This constitutes a 4% and 5% increase respectively. This change in skewness also increases the probability of an individual finding himself on the lower outskirts of the health distribution. These results thus indicate that the nature of the association of income with health beyond the mean, with the risk of very low health scores - indicating severe sickness - driven not only by a deteriorating mean but also by a higher standard deviation and a less left-skewed distribution.

As indicated by the higher order moments, the increase in the health risks are higher when directing the focus further towards the lower end of the health spectrum. Considering $\mathcal{R}_{C0.50}$ for males, we still find a moderate change in the risk from 0.70 to 0.65, constituting a decrease of 6%. For women, we see a decrease from 0.70 to 0.65, i.e. by 7%. This change can be seen as of a similar magnitude as R_{M3} and also similar to the relative change observed for the expected outcome (see above). The relative difference increases to 20% (0.27 to 0.22) and 23% (0.27 to 0.21) for men and women respectively, when considering $\mathcal{R}_{C0.20}$. The greatest relative effect is seen for $\mathcal{R}_{C0.05}$, which sees the risk of falling into the lowest health quintile of the population at 0.06 for "average Joe" and 0.05 for "average Jane" at 15,000€, whereas having an income twice as high reduces that risk down to 0.03 for both, a decrease of 39% and 42% respectively. In other words, the risk of extremely bad health can be roughly halved by doubling the net equivalent income from 15,000€to 30,000€. Obviously, the magnitude of this effects is structurally different from the observed 3% increase observed for expected health.

3.5 Reconsidering the Angle of the Health Assessment Perspective

Considering the whole conditional health distribution and changes thereof over the covariate space thus yields potentially starkly different magnitudes for the assessment of the association between income and health. The differences are summarised in Table 5. The relative difference is the absolute difference divided by the measure for 15,000€.

The table shows that the association becomes significantly greater if we focus on the lower health spectrum, with the mean portraying a measure of around 3% while a heavy focus on the lower spectrum by $\mathcal{R}_{M1}$ and $\mathcal{R}_{C0.05}$ yield differences in the order of 39%-42%, i.e. more than tenfold in terms of magnitude.

The conventional perspective generates significant results that allow us to infer the existence of a relationship between income and health. How, then, does our more complicated statistical artillery help us, beyond the results more easily generated using well-established mean-based analyses?

	males		
	15,000€	30,000€	Difference
$\mathcal{R}_{M3}$	0.67[0.67; 0.68]	0.61[0.60; 0.61]	10.11%[10.01%;10.20%]
$\mathcal{R}_{M2}$	0.20[0.20; 0.21]	0.14[0.13; 0.14]	33.64%[33.41%;33.87%]
$\mathcal{R}_{M1}$	0.04[0.04; 0.05]	0.03[0.03; 0.03]	39.21%[38.77%;39.71%]
μ	45.33[43.84;46.69]	46.65[45.11;47.97]	2.90%[2.27%; 3.40%]
$\mathcal{R}_{C0.50}$	0.68[0.63; 0.74]	0.65[0.59; 0.71]	5.53%[3.75%; 7.83%]
$\mathcal{R}_{C0.20}$	0.27[0.22; 0.33]	0.22[0.17; 0.28]	19.92%[15.14%;25.91%]
$\mathcal{R}_{C0.05}$	0.06[0.03; 0.08]	0.03[0.02; 0.06]	38.98%[30.05%;48.95%]
	females		
	15,000€	30,000€	Difference
$\mathcal{R}_{M3}$	0.58[0.58; 0.58]	0.50[0.50; 0.50]	13.27%[13.15%;13.39%]
$\mathcal{R}_{M2}$	0.22[0.21; 0.22]	0.15[0.15; 0.15]	29.54%[29.32%;29.80%]
$\mathcal{R}_{M1}$	0.04[0.04; 0.05]	0.03[0.03; 0.03]	40.38%[39.96%;40.89%]
μ	45.09[43.69;46.36]	46.63[45.19;47.87]	3.43%[2.82%: 4.01%]
$\mathcal{R}_{C0.50}$	0.70[0.65; 0.75]	0.65[0.60; 0.71]	7.24%[5.14%; 9.48%]
$\mathcal{R}_{C0.20}$	0.27[0.23; 0.33]	0.21[0.17; 0.27]	22.83%[17.88%;28.13%]
$\mathcal{R}_{C0.05}$	0.05[0.04; 0.08]	0.03[0.02; 0.05]	42.36%[33.98%;50.58%]

Table 3: Seven measures on the health-income association.

The answer to this important question lies in the fact that while average population health is an important construct for many purposes, we cannot properly calculate the utility of alternative distributions of health using only this summary statistic. This is because the utility function for health is generally thought to be concave. If the utility gain from increases in health status at the low end of the health spectrum is greater than at the high end, changes to the distribution of income that do not affect the mean health of the population but lessen the number of people in very poor health would nevertheless be preferable at a societal level. At a policy level, too, there are good reasons to care at least as much about the risk of people being in poor health as about average health achievement in the population, since the primary purpose of public or private health insurance is to cover the cost of caring for those who are ill, rather than focussing on improving the health of the already healthy even further. When we think about the relationship between health and income, then, we want to be able to pay attention not only to the average effect of income on health, but also to the where in the health distribution people of various incomes are more likely to fall. That is what SADR allows us to do. The results we have shown here demonstrate that given the significantly greater likelihood of being in bad health at lower income levels, that while the income-health relation (focussing on average health) may not be of great magnitude, the income-illness relation (concentrating on the ill) certainly is of considerable magnitude.

4 Conclusion

In this paper, we have shown that generalized linear models, whether they deal with (coerced) binary responses or continuous responses, focus on the conditional expectation. Yet, we argue that where the distribution of health outcome is more complex it is often insufficient to solely look at the expectation. For this reason we propose a distributional perspective that allows for a focus on risks regarding the lower end of the full conditional health distribution. Using structured additive distributional regression we show that it is possible to estimate such full conditional health distributions for both multicategorical and continuous measures of health outcomes. Looking at health data from the German Socio-Economic Panel, we find that the standard expectation-based perspective may neglect potentially important aspects of the relationship between health and income. In particular, we show that the risk of being in very poor health is much more strongly related to income than the average health status. We find that the risk for the "average Joe" and "average Jane" of belonging to the severely sick population increases between 39% and 42% when the net equivalent household income is changed from the median income of the poorer half of the population (15,000€) to the median income of the richer half (30,000€) in Germany. This exceeds the income-related change in average health status that is estimated using standard estimation techniques by more than tenfold. This suggests that mean-based perspectives may underestimate the effect of changes in the income distribution on well-being (given a concave health-utility relationship) and/or on health care expenditures (given that health care is more cost intensive at the lower end of the health distribution).

Based on the findings of this paper, we propose that future estimates of the health-income relationship take into account not only the mean reported health (or the probability of a dichotomized health measure in an income group), but also employ risk measures focusing on very poor health outcomes like the the ones used in this paper. Not only would this put more emphasis on the the lower end of the spectrum, where we argue it is merited. In addition to addressing problems associated with non-linearities with respect to well-being and/or health care and mean regression (see above), a distributional approach and risk-based measures may also unify the interpretation of the otherwise starkly different results that can arise depending on whether discrete data (and odds-ratios) or continuous data (and arithmetic means) are used for the assessment. We find that using SADR, the estimated magnitude of the income-health relationship is very similar for the single-item self-rated health measure and the SF-12. The distributional approach thus may contribute to the convergence of findings from the epidemiological literature (which mainly employs discrete measures like self-rated health and odds ratios) and the health economics literature (which tends to employ continuous measures like the SF-12 and arithmetic means).

Several extensions to the present approach might be considered. One particularly interesting modification to model the full joint distribution of health and income with respect to other covariates such as age and education. This would be feasible by applying bivariate structured additive distributional regression, which uses copula structures to model the interrelations of the dependent variables (see Klein et al., forthcoming) and would allow for the construction of conditional concentration curves across the covariate space. While technically challenging, this approach would not only incorporate the workhorse method in the health economics literature into the proposed framework, but would also allow researchers to consider distributional aspects beyond the mean without the need to define threshold values. Such advancements are needed because, to paraphrase Thomas Piketty (2014), failing to deal with the distributional nature of the health-income relationship rarely serves the interests of the least well-off.

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A Appendix

A.1 Data

As primary source for our data, we use the SOEP database (Wagner et al., 2007). We use all available samples in 2012, i.e. samples A-L. Concerning the wave, we only consider the wave from 2012, i.e. wave BC, for all questions on current status. For questions asked with respect to the whole last year, we consider questions from 2013, referring back to 2012, i.e. questions from wave BD. Only taking those values for which we have the full set of variables, as described below, this yields 16,732 observations (7,820 males and 8,903 females).

As for the dependent variable we simply take the single item self-rate health response (`bcp91`) on the one hand. On the other hand, we consider the physical condition score from the SF-12 (`PCS`) which is directly available via the `HEALTH` file in the SOEP.

As a variable for income we use the household's net income as the base (`i1110213` from the `BCPEQUIV` file) and divide it by the equivalised household size, based on the OECD equivalence scale (using the variables `bchhgr` and `bckzahl` from the `BCKIND` file). Thereby the first adult is given a weight of 1, whereas to every additional person aged 14 and over is given the weight of 0.5. Each child aged 13 and under is given a weight of 0.3. Each individual living in the household is then given the household's net equivalent income.

For the explanatory variable age, we simply use the year of birth (`gebjahr`) and subtract it from 2012, while the sex is determined by the variable `bcsex`.

The education level is taken on grounds of the variable `ISCED12` (from the person-related status and generated variables `PGEN`). All observations equal or lower than 5 (higher vocational training) are put in the category no higher education with only those persons with a value of 6 (higher education) considered for the category higher education.

The nationality is obtained directly from the SOEP based on the person's contemporary status (`BCP139`).

The marrital status is taken from the 6 item response to the family status available in the SOEP (`BCP129`), which is reduced to four categories as described in the text.

For the spatial effect we use the variable `bcbula` with the variable *east* set to unity for all federal states formerly belonging to the German Democratic Republic, including the whole of Berlin. West Berlin (as defined prior to 1990) is not accounted for in our sample and treated like a state from the former East.

A.2 Model Selection

For model selection, we use the DIC which has been shown to work well for distributional regression models (Klein et al., 2015). As the primary focus of this paper is on the existence of covariate effects beyond the mean, we focus solely on the comparison of three regression models and their distributional assumptions.[6] A comprehensive model selection analysis is beyond the scope of this paper and thus we confine ourselves to a comparison of the following three models:

- $M1$ As benchmark model, we consider a homoskedastic, gaussian model. In this model the focus is solely directed towards the expectation (μ), with the other parameter (σ^2) considered a nuisance parameter and set as a constant. This is the standard assumption used for most generalised linear models employed in the literature on the health-income relation.
- $M2$ As a second model, we consider a heteroskedastic, gaussian model. In this model the variance is now no longer considered a constant as we explicitly allow the standard deviation of the normal distribution to vary across the covariate space. While this already considerably enhances flexibility, the normal distribution is by definition symmetric such that it does not allow for the modelling of changing skewness over the covariate space.
- $M3$ As a third model, we consider a two parameter gamma distribution with both parameters allowed to vary across the covariate space. In contrast to the normal distribution, the gamma distribution is not confined to a symmetric form and varies its skewness in relation to its scale parameter.

	Modelassumption	male	female
$M1$	$\mathcal{N}(\mu$ varying, σ^2 constant)	19871.1	23154.2
$M2$	$\mathcal{N}(\mu$ varying, σ^2 varying)	19484.2	22881.6
$M3$	Γ (μ varying, σ varying)	18977.6	22492.3

Table 4: DIC results on distributional assumptions for PCS.

The resultant DICs for these three models are displayed in Table 4. The DICs displayed in Table 4 indicate that out of the three distributions the gamma distribution ($M3$) is the best suited distribution, as it has the lowest DIC in all four cases. However, it should be re-emphasised, that the choice of the models here can only be seen as preliminary and further research must be done on the choice of adequate parametric conditional health distributions. Thus, we will consider only the gamma distribution for our assessment of the relation between income and physical health.

[6] We thus do not consider the issues of variable selection and simply rely on the variable selection proposed by Lorgelly and Lindley (2008) to be adequate for the construction of a generic predictor applied for all parameters.

A.3 Random Effects

Table 5 displays the random effects for the individual federal states for the multinomial. The federal states are are abbreviated according to the abbreviations for regions at the EU level[7].
As one can observe there are significant changes associated with the different states, which is in line with the literature on regional health differences (Eibich and Ziebarth, 2014).

	$\eta^{\tilde{\pi}_1}$	$\eta^{\tilde{\pi}_2}$	$\eta^{\tilde{\pi}_3}$	$\eta^{\tilde{\pi}_4}$
males				
SH	-0.229[-0.413;-0.036]	0.113[0.008; 0.225]	0.001[-0.120; 0.135]	-0.057[-0.323; 0.228]
HH	-0.212[-0.390;-0.020]	-0.247[-0.352;-0.135]	-0.058[-0.177; 0.074]	0.082[-0.178; 0.359]
NI	-0.111[-0.296; 0.080]	-0.106[-0.207; 0.002]	0.347[0.230; 0.480]	-0.437[-0.689;-0.151]
HB	0.846[0.664; 1.036]	0.122[0.018; 0.233]	-0.498[-0.618;-0.363]	0.558[0.303; 0.850]
NW	0.091[-0.092; 0.283]	-0.005[-0.105; 0.103]	0.097[-0.019; 0.226]	-0.180[-0.430; 0.100]
HE	-0.170[-0.353; 0.021]	-0.043[-0.145; 0.065]	0.076[-0.041; 0.206]	-0.043[-0.294; 0.233]
RP	0.180[-0.001; 0.374]	0.076[-0.024; 0.185]	-0.059[-0.181; 0.073]	-0.226[-0.486; 0.056]
BW	-0.225[-0.407;-0.033]	0.003[-0.098; 0.112]	0.014[-0.102; 0.144]	-0.546[-0.798;-0.271]
BY	-0.169[-0.350; 0.021]	-0.124[-0.225;-0.015]	0.179[0.059; 0.309]	0.351[0.101; 0.631]
SL	0.558[0.381; 0.751]	0.478[0.380; 0.590]	0.143[0.026; 0.269]	1.114[0.858; 1.390]
BE	0.085[-0.107; 0.272]	0.135[0.017; 0.245]	0.063[-0.078; 0.199]	0.082[-0.211; 0.364]
BB	0.185[-0.016; 0.375]	-0.131[-0.250;-0.020]	0.245[0.101; 0.381]	0.085[-0.211; 0.373]
MV	0.223[0.019; 0.416]	0.103[-0.016; 0.216]	-0.206[-0.346;-0.068]	0.060[-0.231; 0.348]
SN	0.209[0.010; 0.398]	0.038[-0.079; 0.152]	0.051[-0.092; 0.186]	-0.402[-0.691;-0.119]
ST	-0.210[-0.413;-0.013]	0.237[0.118; 0.349]	0.173[0.032; 0.308]	0.541[0.249; 0.825]
TH	0.124[-0.079; 0.321]	-0.076[-0.194; 0.037]	-0.048[-0.186; 0.090]	0.329[0.042; 0.618]
females				
	$\eta^{\tilde{\pi}_1}$	$\eta^{\tilde{\pi}_2}$	$\eta^{\tilde{\pi}_3}$	$\eta^{\tilde{\pi}_4}$
SH	-0.285[-0.473;-0.091]	-0.168[-0.271;-0.056]	0.001[-0.078; 0.082]	0.078[-0.113; 0.276]
HH	-0.005[-0.190; 0.195]	-0.309[-0.413;-0.197]	-0.041[-0.118; 0.043]	-0.014[-0.202; 0.190]
NI	0.216[0.026; 0.412]	0.170[0.069; 0.277]	-0.086[-0.163;-0.005]	0.215[0.031; 0.416]
HB	0.275[0.087; 0.471]	0.051[-0.051; 0.165]	0.182[0.104; 0.267]	0.306[0.114; 0.518]
NW	0.175[-0.013; 0.375]	0.171[0.070; 0.277]	-0.050[-0.128; 0.033]	-0.036[-0.225; 0.162]
HE	0.126[-0.061; 0.324]	0.095[-0.007; 0.202]	-0.044[-0.120; 0.038]	-0.246[-0.432;-0.050]
RP	-0.040[-0.226; 0.160]	0.094[-0.007; 0.204]	0.169[0.093; 0.253]	-0.376[-0.567;-0.168]
BW	0.317[0.131; 0.514]	-0.095[-0.195; 0.015]	-0.075[-0.152; 0.006]	-0.046[-0.234; 0.157]
BY	-0.130[-0.316; 0.066]	0.072[-0.029; 0.181]	0.000[-0.079; 0.082]	0.135[-0.055; 0.335]
SL	-0.125[-0.306; 0.074]	0.154[0.055; 0.265]	0.066[-0.013; 0.146]	0.358[0.169; 0.554]
BE	-0.320[-0.517;-0.126]	0.313[0.195; 0.424]	-0.093[-0.189;-0.003]	0.755[0.528; 0.968]
BB	0.230[0.024; 0.421]	-0.177[-0.295;-0.068]	0.037[-0.066; 0.128]	0.151[-0.081; 0.365]
MV	-0.182[-0.389; 0.017]	0.230[0.114; 0.341]	0.267[0.164; 0.359]	-0.520[-0.751;-0.300]
SN	-0.131[-0.332; 0.068]	0.128[0.011; 0.238]	0.118[0.017; 0.209]	-0.005[-0.232; 0.210]
ST	0.997[0.792; 1.200]	-0.130[-0.247;-0.019]	-0.004[-0.104; 0.086]	-0.141[-0.372; 0.070]
TH	-0.024[-0.230; 0.174]	-0.092[-0.209; 0.018]	-0.178[-0.278;-0.087]	0.188[-0.038; 0.400]

Table 5: Random effects for federal states on $\eta^{\tilde{\pi}_1}$,$\eta^{\tilde{\pi}_2}$,$\eta^{\tilde{\pi}_3}$ and $\eta^{\tilde{\pi}_4}$ for SRH.

Table 6 displays the random effects for the parameters of the gamma distribution.
As for the multinomial case, we can observe significant health variations for both parameters

[7]See http://www.bmelv-statistik.de/de/daten-tabellen-suche/abkuerzungen-der-bundeslaender/.

for some federal states, although in case of σ significance is restricted to Hesse and Baden-Wuerttemberg for males alone.

	males		**females**	
	η^{μ}	η^{σ}	η^{μ}	η^{σ}
SH	0.011[-0.010; 0.031]	-0.009[-0.161; 0.160]	0.008[-0.012; 0.028]	-0.008[-0.092; 0.070]
HH	-0.025[-0.049;-0.003]	-0.045[-0.221; 0.116]	-0.017[-0.039; 0.003]	0.019[-0.055; 0.128]
NI	0.000[-0.019; 0.017]	-0.046[-0.162; 0.080]	0.005[-0.013; 0.021]	-0.025[-0.110; 0.036]
HB	-0.003[-0.025; 0.022]	-0.119[-0.335; 0.046]	-0.003[-0.026; 0.020]	-0.009[-0.102; 0.071]
NW	0.007[-0.011; 0.020]	-0.022[-0.123; 0.095]	0.012[-0.004; 0.026]	-0.007[-0.070; 0.056]
HE	0.000[-0.019; 0.016]	0.135[0.013; 0.294]	0.000[-0.017; 0.016]	0.029[-0.033; 0.124]
RP	0.009[-0.010; 0.028]	0.024[-0.111; 0.164]	0.003[-0.015; 0.021]	-0.017[-0.099; 0.046]
BW	-0.010[-0.026; 0.004]	0.219[0.104; 0.352]	-0.007[-0.023; 0.006]	0.025[-0.036; 0.109]
BY	-0.006[-0.023; 0.009]	0.017[-0.095; 0.134]	-0.002[-0.017; 0.013]	0.013[-0.047; 0.081]
SL	0.019[-0.006; 0.047]	-0.147[-0.369; 0.027]	0.002[-0.021; 0.026]	-0.017[-0.125; 0.062]
BE	-0.002[-0.024; 0.020]	-0.127[-0.297; 0.043]	0.001[-0.021; 0.020]	-0.001[-0.078; 0.082]
BB	0.002[-0.019; 0.023]	0.095[-0.057; 0.263]	0.002[-0.017; 0.022]	0.016[-0.061; 0.115]
MV	0.003[-0.021; 0.025]	0.068[-0.095; 0.272]	0.011[-0.012; 0.033]	-0.001[-0.082; 0.082]
SN	-0.001[-0.022; 0.019]	-0.058[-0.203; 0.074]	-0.001[-0.022; 0.019]	-0.006[-0.088; 0.064]
ST	0.007[-0.014; 0.031]	-0.013[-0.182; 0.149]	0.003[-0.018; 0.026]	-0.017[-0.121; 0.061]
TH	-0.008[-0.030; 0.015]	0.034[-0.134; 0.210]	-0.015[-0.036; 0.007]	0.010[-0.068; 0.107]

Table 6: Random effects for federal states on η^{μ} and η^{σ} for PCS.

A.4 Other Covariate Combinations

In this section we show the seven health measures displayed in Section 3.5 for "average Joe" and "average Jane" for a different set of characteristics. For the sake of brevity and simplicity, we constrain the sets considered to 7 different sets, always varying only one covariate while keeping all the other covariates at the values used for "average Joe" and "average Jane".

Tables 7 and 8 display the seven health measures for two other ages, namely the first and the third quartile of the ages in the sample: 40 years and 66 years, respectively.
As can be seen from the tables, the general structure persists, whereby the differences between the health measures for the two different income levels becomes more pronounced as the focus is shifted towards the lower end of the health spectrum. Moreover, one may note that the health situation is generally better for younger individuals than for older individuals, which is to be expected given the physical deterioration of the body as part of the ageing process.

Table 9 displays the seven health measures for non-German nationals. Again the basic pattern remains such that income related differences are more more pronounced (in relative terms) in the lower end of the health spectrum. One other thing which can be observed from the table is the lower health risks and slightly better average health enjoyed by non-German nationals, which is in line with the healthy-migrant effect found in the literature (see Bjornstrom and Kuhl, 2014).

Table 10 displays the seven health measures for individuals in the fourth education level, i.e. those individuals with higher education. Again, the basic pattern remains in the sense that income related differences are significantly higher when focussing on the lower end of the health spectrum. As one would expect, higher education levels additionally mean a higher average health outcome and lower risk measures for both men and women, c.p.

Table 11 displays the seven health measures for individuals in the third marrital status, i.e. those individuals who are sinlge, which is the second most frequent observed marrital status in our sample. As above, the basic pattern is such that income related differences are significantly higher when looking at the lower end of the health spectrum. In terms of the absolute levels, we observe a slightly lower average health and slightly elevated risks, which is reasonable given the positive health effects that stable relationships are thought to have.

Tables 12 and 13 display the seven health measures for two other federal states in Germany, namely BadenWurttemberg and Mecklenburg-Western Pomerania. BadenWurttemberg is a very wealth state in the South-West of Germany while Mecklenburg-Western Pomerania is an economically rather depressed state in the North-East of Germany. As can be seen from the tables, the general structure persists again, i.e. the differences between the health measures for the two different income levels becomes more pronounced as the focus is shifted towards the lower end of the health spectrum. As one would expect the wealthier federal state BadenWurttemberg also features better health measures than the poorer Mecklenburg-Western Pomerania, which is to be expected given the positive effects of gdp on state finances and thus available funds for the health infrastructure in these regions.

	males		
	15,000€	30,000€	Difference
$\mathcal{R}_{M3}$	0.53[0.53; 0.53]	0.46[0.45; 0.46]	13.94%[13.81%;14.07%]
$\mathcal{R}_{M2}$	0.14[0.14; 0.14]	0.09[0.09; 0.09]	37.39%[37.14%;37.63%]
$\mathcal{R}_{M1}$	0.03[0.03; 0.03]	0.02[0.01; 0.02]	42.79%[42.34%;43.30%]
μ	48.95[47.54;50.15]	50.18[48.72;51.36]	2.51%[2.94%; 1.96%]
$\mathcal{R}_{C0.50}$	0.55[0.49; 0.61]	0.49[0.43; 0.56]	10.30%[7.13%;13.42%]
$\mathcal{R}_{C0.20}$	0.14[0.10; 0.19]	0.10[0.07; 0.14]	30.14%[22.74%;37.99%]
$\mathcal{R}_{C0.05}$	0.02[0.01; 0.03]	0.01[0.00; 0.02]	52.95%[40.94%;63.24%]
	females		
	15,000€	30,000€	Difference
$\mathcal{R}_{M3}$	0.46[0.46; 0.46]	0.38[0.38; 0.39]	16.65%[16.51%;16.79%]
$\mathcal{R}_{M2}$	0.15[0.15; 0.15]	0.10[0.10; 0.10]	33.38%[33.13%;33.64%]
$\mathcal{R}_{M1}$	0.02[0.02; 0.03]	0.01[0.01; 0.01]	44.05%[43.60%;44.56%]
μ	48.50[47.12;49.65]	49.95[48.57;51.09]	2.99%[3.50%; 2.45%]
$\mathcal{R}_{C0.50}$	0.57[0.51; 0.63]	0.50[0.44; 0.57]	11.61%[8.65%;14.64%]
$\mathcal{R}_{C0.20}$	0.15[0.12; 0.20]	0.11[0.08; 0.15]	31.01%[24.75%;37.99%]
$\mathcal{R}_{C0.05}$	0.02[0.01; 0.03]	0.01[0.00; 0.02]	52.86%[42.70%;61.48%]

Table 7: Seven measures on the health-income association for 40yrs of age (all other covariates the same).

	males		
	15,000€	30,000€	Difference
$\mathcal{R}_{M3}$	0.77[0.77; 0.77]	0.72[0.71; 0.72]	7.21%[7.13%; 7.28%]
$\mathcal{R}_{M2}$	0.27[0.27; 0.27]	0.19[0.18; 0.19]	30.32%[30.12%;30.56%]
$\mathcal{R}_{M1}$	0.06[0.06; 0.06]	0.04[0.04; 0.04]	36.05%[35.63%;36.56%]
μ	41.10[39.48;42.51]	42.49[40.91;43.93]	3.38%[4.04%; 2.70%]
$\mathcal{R}_{C0.50}$	0.81[0.76; 0.85]	0.78[0.73; 0.84]	2.71%[1.47%; 4.24%]
$\mathcal{R}_{C0.20}$	0.43[0.37; 0.49]	0.37[0.32; 0.44]	12.99%[9.53%;17.13%]
$\mathcal{R}_{C0.05}$	0.13[0.09; 0.18]	0.09[0.06; 0.13]	29.29%[22.27%;37.06%]
	females		
	15,000€	30,000€	Difference
$\mathcal{R}_{M3}$	0.70[0.70; 0.70]	0.63[0.63; 0.63]	9.66%[9.56%; 9.76%]
$\mathcal{R}_{M2}$	0.31[0.30; 0.31]	0.23[0.23; 0.23]	25.22%[25.02%;25.46%]
$\mathcal{R}_{M1}$	0.07[0.07; 0.07]	0.04[0.04; 0.05]	36.48%[36.05%;36.99%]
μ	40.83[39.25;42.18]	42.49[40.95;43.83]	4.07%[4.77%; 3.36%]
$\mathcal{R}_{C0.50}$	0.83[0.79; 0.87]	0.80[0.75; 0.84]	3.73%[2.43%; 5.35%]
$\mathcal{R}_{C0.20}$	0.44[0.38; 0.50]	0.37[0.31; 0.43]	15.70%[12.28%;19.74%]
$\mathcal{R}_{C0.05}$	0.12[0.09; 0.16]	0.08[0.06; 0.12]	33.37%[26.40%;40.95%]

Table 8: Seven measures on the health-income association for 66yrs of age (all other covariates the same).

	males		
	15,000€	30,000€	Difference
$\mathcal{R}_{M3}$	0.55[0.55; 0.55]	0.48[0.47; 0.48]	13.67%[13.54%;13.80%]
$\mathcal{R}_{M2}$	0.17[0.17; 0.17]	0.11[0.11; 0.11]	36.18%[35.92%;36.45%]
$\mathcal{R}_{M1}$	0.03[0.03; 0.03]	0.02[0.02; 0.02]	41.70%[41.21%;42.19%]
μ	46.39[44.77;47.91]	47.67[45.96;49.12]	2.75%[3.24%; 2.17%]
$\mathcal{R}_{C0.50}$	0.65[0.59; 0.72]	0.61[0.54; 0.69]	6.63%[4.53%; 9.52%]
$\mathcal{R}_{C0.20}$	0.23[0.17; 0.29]	0.17[0.12; 0.23]	23.44%[17.16%;30.42%]
$\mathcal{R}_{C0.05}$	0.04[0.02; 0.06]	0.02[0.01; 0.04]	43.76%[33.40%;55.07%]
	females		
	15,000€	30,000€	Difference
$\mathcal{R}_{M3}$	0.56[0.56; 0.56]	0.48[0.48; 0.49]	13.62%[13.49%;13.76%]
$\mathcal{R}_{M2}$	0.28[0.28; 0.29]	0.21[0.20; 0.21]	26.99%[26.74%;27.24%]
$\mathcal{R}_{M1}$	0.05[0.05; 0.05]	0.03[0.03; 0.03]	38.68%[38.21%;39.21%]
μ	44.34[42.71;45.83]	45.92[44.27;47.32]	3.55%[4.11%; 2.90%]
$\mathcal{R}_{C0.50}$	0.74[0.68; 0.79]	0.69[0.62; 0.75]	6.52%[4.50%; 8.94%]
$\mathcal{R}_{C0.20}$	0.30[0.24; 0.36]	0.23[0.17; 0.30]	22.57%[17.10%;28.81%]
$\mathcal{R}_{C0.05}$	0.06[0.03; 0.09]	0.03[0.02; 0.06]	43.04%[33.41%;52.54%]

Table 9: Seven measures on the health-income association for non-German nationals (all other covariates the same).

	males		
	15,000€	30,000€	Difference
$\mathcal{R}_{M3}$	0.62[0.62; 0.62]	0.55[0.54; 0.55]	11.60%[11.48%;11.69%]
$\mathcal{R}_{M2}$	0.20[0.20; 0.20]	0.13[0.13; 0.13]	34.19%[33.94%;34.41%]
$\mathcal{R}_{M1}$	0.05[0.05; 0.05]	0.03[0.03; 0.03]	39.59%[39.14%;40.04%]
μ	46.16[44.83;47.44]	47.41[46.12;48.71]	2.71%[3.29%; 2.19%]
$\mathcal{R}_{C0.50}$	0.65[0.60; 0.70]	0.61[0.55; 0.67]	6.47%[4.36%; 8.67%]
$\mathcal{R}_{C0.20}$	0.24[0.20; 0.29]	0.19[0.15; 0.24]	21.43%[16.60%;27.13%]
$\mathcal{R}_{C0.05}$	0.05[0.03; 0.07]	0.03[0.02; 0.04]	41.04%[32.26%;50.20%]
	females		
	15,000€	30,000€	Difference
$\mathcal{R}_{M3}$	0.51[0.51; 0.52]	0.44[0.43; 0.44]	15.13%[15.00%;15.24%]
$\mathcal{R}_{M2}$	0.16[0.16; 0.17]	0.11[0.11; 0.11]	32.17%[31.93%;32.40%]
$\mathcal{R}_{M1}$	0.03[0.03; 0.04]	0.02[0.02; 0.02]	42.60%[42.15%;43.06%]
μ	46.34[45.08;47.53]	47.83[46.60;49.05]	3.22%[3.81%; 2.66%]
$\mathcal{R}_{C0.50}$	0.64[0.60; 0.69]	0.59[0.54; 0.64]	8.21%[6.20%;10.52%]
$\mathcal{R}_{C0.20}$	0.24[0.20; 0.28]	0.18[0.15; 0.22]	23.80%[19.28%;28.84%]
$\mathcal{R}_{C0.05}$	0.05[0.03; 0.06]	0.03[0.02; 0.04]	42.52%[34.90%;50.27%]

Table 10: Seven measures on the health-income association for the fourth education level (all other covariates the same).

	males		
	15,000€	30,000€	Difference
$\mathcal{R}_{M3}$	0.73[0.73; 0.73]	0.67[0.67; 0.67]	8.20%[8.11%; 8.29%]
$\mathcal{R}_{M2}$	0.23[0.23; 0.24]	0.16[0.16; 0.16]	31.79%[31.57%;32.03%]
$\mathcal{R}_{M1}$	0.07[0.06; 0.07]	0.04[0.04; 0.04]	37.03%[36.59%;37.53%]
μ	45.14[43.59;46.61]	46.42[44.85;47.91]	2.85%[3.45%; 2.27%]
$\mathcal{R}_{C0.50}$	0.68[0.62; 0.73]	0.65[0.58; 0.70]	5.32%[3.47%; 7.46%]
$\mathcal{R}_{C0.20}$	0.29[0.23; 0.34]	0.23[0.18; 0.29]	18.82%[13.69%;24.39%]
$\mathcal{R}_{C0.05}$	0.07[0.04; 0.10]	0.04[0.02; 0.07]	36.79%[27.47%;46.40%]
	females		
	15,000€	30,000€	Difference
$\mathcal{R}_{M3}$	0.64[0.64; 0.64]	0.57[0.56; 0.57]	11.34%[11.23%;11.46%]
$\mathcal{R}_{M2}$	0.27[0.26; 0.27]	0.19[0.19; 0.20]	27.08%[26.86%;27.34%]
$\mathcal{R}_{M1}$	0.06[0.06; 0.07]	0.04[0.04; 0.04]	37.87%[37.41%;38.37%]
μ	45.07[43.65;46.43]	46.60[45.13;48.03]	3.39%[3.97%; 2.83%]
$\mathcal{R}_{C0.50}$	0.69[0.64; 0.74]	0.65[0.58; 0.70]	6.94%[5.02%; 9.37%]
$\mathcal{R}_{C0.20}$	0.28[0.23; 0.33]	0.22[0.17; 0.28]	21.91%[16.96%;27.61%]
$\mathcal{R}_{C0.05}$	0.06[0.04; 0.08]	0.03[0.02; 0.06]	41.03%[32.39%;49.66%]

Table 11: Seven measures on the health-income association for the third marrital status (all other covariates the same).

		males	
	15,000€	30,000€	Difference
$\mathcal{R}_{M3}$	0.60[0.59; 0.60]	0.52[0.52; 0.53]	12.50%[12.37%;12.62%]
$\mathcal{R}_{M2}$	0.17[0.17; 0.18]	0.11[0.11; 0.11]	35.70%[35.44%;35.98%]
$\mathcal{R}_{M1}$	0.04[0.04; 0.04]	0.02[0.02; 0.03]	40.93%[40.47%;41.44%]
μ	47.24[45.82;48.69]	48.48[47.06;49.95]	2.64%[3.19%; 2.06%]
$\mathcal{R}_{C0.50}$	0.61[0.55; 0.67]	0.56[0.50; 0.63]	7.33%[5.11%;10.10%]
$\mathcal{R}_{C0.20}$	0.21[0.16; 0.26]	0.16[0.11; 0.21]	23.18%[17.35%;29.86%]
$\mathcal{R}_{C0.05}$	0.04[0.02; 0.06]	0.02[0.01; 0.04]	42.91%[33.07%;53.63%]
		females	
	15,000€	30,000€	Difference
$\mathcal{R}_{M3}$	0.55[0.55; 0.56]	0.48[0.47; 0.48]	14.07%[13.95%;14.22%]
$\mathcal{R}_{M2}$	0.20[0.20; 0.21]	0.14[0.14; 0.14]	30.39%[30.17%;30.69%]
$\mathcal{R}_{M1}$	0.04[0.04; 0.04]	0.02[0.02; 0.02]	41.24%[40.84%;41.82%]
μ	46.46[45.10;47.83]	47.95[46.61;49.34]	3.22%[3.83%; 2.66%]
$\mathcal{R}_{C0.50}$	0.65[0.59; 0.71]	0.59[0.53; 0.66]	8.68%[6.41%;11.35%]
$\mathcal{R}_{C0.20}$	0.22[0.18; 0.27]	0.17[0.12; 0.21]	25.73%[20.76%;31.82%]
$\mathcal{R}_{C0.05}$	0.04[0.02; 0.05]	0.02[0.01; 0.03]	45.89%[37.79%;54.90%]

Table 12: Seven measures on the health-income association for BadenWurttemberg (all other covariates the same).

		males	
	15,000€	30,000€	Difference
$\mathcal{R}_{M3}$	0.63[0.63; 0.63]	0.56[0.56; 0.56]	11.53%[11.45%;11.64%]
$\mathcal{R}_{M2}$	0.24[0.24; 0.24]	0.16[0.16; 0.16]	32.72%[32.52%;32.95%]
$\mathcal{R}_{M1}$	0.04[0.04; 0.04]	0.02[0.02; 0.02]	38.73%[38.32%;39.26%]
μ	45.89[44.67;47.15]	47.17[45.91;48.38]	2.78%[3.33%; 2.23%]
$\mathcal{R}_{C0.50}$	0.66[0.61; 0.71]	0.62[0.57; 0.67]	6.05%[4.19%; 8.06%]
$\mathcal{R}_{C0.20}$	0.25[0.21; 0.30]	0.20[0.16; 0.25]	20.59%[15.86%;25.92%]
$\mathcal{R}_{C0.05}$	0.05[0.03; 0.08]	0.03[0.02; 0.05]	39.65%[31.28%;48.80%]
		females	
	15,000€	30,000€	Difference
$\mathcal{R}_{M3}$	0.67[0.66; 0.67]	0.59[0.59; 0.60]	10.80%[10.71%;10.89%]
$\mathcal{R}_{M2}$	0.24[0.23; 0.24]	0.17[0.17; 0.17]	27.97%[27.77%;28.22%]
$\mathcal{R}_{M1}$	0.05[0.05; 0.06]	0.03[0.03; 0.03]	38.76%[38.35%;39.27%]
μ	45.24[44.03;46.44]	46.78[45.54;47.95]	3.40%[4.01%; 2.79%]
$\mathcal{R}_{C0.50}$	0.69[0.65; 0.74]	0.64[0.59; 0.69]	7.27%[5.35%; 9.29%]
$\mathcal{R}_{C0.20}$	0.27[0.23; 0.32]	0.21[0.17; 0.25]	22.87%[18.54%;27.65%]
$\mathcal{R}_{C0.05}$	0.05[0.04; 0.07]	0.03[0.02; 0.05]	41.92%[34.65%;50.19%]

Table 13: Seven measures on the health-income association for Mecklenburg-Western Pomerania (all other covariates the same).