Unemployment benefit reforms, distribution and efficiency

Irene Endres[(a)], Andrey Launov[(a,b)] and Klaus Wälde[(a,b,c), *]
[(a)]University of Würzburg, [(b)]UC Louvain-la-Neuve, [(c)]University of Glasgow

(Preliminary version. Work still in progress!)

January 8, 2008

Abstract

We formulate an equilibrium search model with stepwise profile of benefit payments and endogenous search effort. Time-dependence of optimal search effort generates an endogenous distribution of unemployment duration characterized by a time-varying hazard function. Applying tools from the literature on semi-Markov processes, we obtain the expression for the expected unemployment rate under heterogeneous search effort. We perform structural estimation of the model using the data of a German labour market and discuss the effects a recent Hartz IV reform. Our results show that although the reform has contributed to the reduction of the aggregate unemployment rate, welfare of both employed and unemployed workers has gone down. The result for the value of a firm is ambiguous. However it is unlikely that the reform leaves a scope for a wealth transfer from firms to workers.

JEL Codes: J65

Keywords: equilibrium search, time-dependent unemployment benefits, structural estimation

*All authors: University of Würzburg, Department of Economics, Sanderring 2, 97070 Würzburg, Germany. Fax: +49.931.888-7025. Contact details: Irene.Endres@Uni-Wuerzburg.de, Phone: +49.931.31-2952. Andrey.Launov@Uni-Wuerzburg.de, Phone: +49.931.31-2953. Klaus@Waelde.com, www.waelde.com. Phone: +49.931.31-2950.

1 Introduction

Unemployment in a number of welfare states has been a problem for many decades now. There seems to be a consensus in the literature that a combination of shocks and institutional arrangements is the main cause of this problem (Ljungqvist and Sargent, 1998, 2008; Blanchard and Wolfers, 2000). Neither institutions nor shocks alone explain the rise in unemployment: institutions have always been there but unemployment has not (at least not at this level) and shocks have hit many countries but not all countries have high unemployment rates. As shocks will not go, it should follow that in order to reduce unemployment one needs to weaken the role of institutions. On the reverse side of this conclusion there is an efficiency-equity tradeoff. Weakening the role of institutions many bring the advantages to the production side of the economy but need not necessarily imply that the aggregate social welfare will go up as a result.

The present paper takes unemployment benefit system for one of the institutions and creates a unified framework for analyzing welfare implications of unemployment benefit reforms. Thereby we answer, to which degree weakening the role of unemployment benefit support, as an institution, is desirable.

The theoretical model of this paper is an estimable equilibrium search model that extends the search and matching framework of Pissarides (1985, 2000) by introducing time-dependent unemployment benefits. Benefit payments in our model have a stepwise profile that depends on the duration of unemployment spell, i.e., once unemployment duration reaches a certain given length, say, a year, the benefit is reduced to a lower level. This setting is typical for the majority of real-world economies. Agents do anticipate the reduction of benefits and are let to choose their search intensity accordingly. As a result, optimal search effort of the agents, and thereby the rate of their exit into employment, becomes a function of the unemployment spell. Our result on time-dependence of search effort is similar to that of Mortensen (1977). Moreover, the supply side of our model is close to the model of van den Berg (1990), who considers similar dynamics of the reservation wage in a nonstationary benefit environment without endogenizing the effort. The key difference of our model from the above two landmarks in the nonstationary search literature is that we assume the individual wage bargaining instead of a given offer distribution. This assumption opens a possibility of formulating the equilibrium search model, whereas the models of Mortensen (1977) and van den Berg (1990) are confined to the supply side only.

Being able to formulate an equilibrium model brings a number of advantages.

First of all, time-dependence of the equilibrium solution for the exit rate out of unemployment leads us to the endogenous distribution of equilibrium unemployment duration. This result is new to the literature. Its particular importance lies in providing us with the direct link between the theory and empirics, as the data for unemployment duration and benefits are available in a variety of surveys. Moreover, the shape of our equilibrium distribution of unemployment duration can be further improved by introduction of a complementary non-benefit effect. Time dependence of this effect adds more flexibility the hazard function, making the theoretical model even more attractive for the estimation. Estimability of the theoretical model means that any aspect of the benefit system reform, mentioned above, can be now consistently quantified and discussed on a macro level. Furthermore, abstracting from quantification of the reform itself, we become able to contribute to the empirical discussion that challenges dependence between unemployment benefits and exit decision in practice (the evidence are contradictory: Hujer and Schneider, 1989 and Arulampalam and Stewart, 1995, give a negative answer; Carling et al., 2001 and Røed and Zhang, 2003, answer positively; we can clarify).

The second advantage of the equilibrium setting is of purely theoretical nature, as it allows solving for the optimal design of the unemployment benefit system. There exists a substantial literature that studies optimal benefit payment schemes under asymmetric information (Shavell and Weiss, 1979; Atkeson and Lucas, 1995; Hopenhayn and Nicolini, 1997). Though, these contributions do not analyze aggregate effects of unemployment payment schemes. Acemoglu and Shimer (1999) do consider a general equilibrium model. But their setting is restricted to time-invariant benefits only. Finally, Cahuc and Lehmann (2000) and Fredriksson and Holmlund (2001) address the optimality of a time-dependent design in a similar Mortensen-Pissarides framework. However, neither of the last two papers provides an estimable model. The work on the optimal design is currently in progress.

Third, endogenous search effort given time-dependent unemployment benefits introduces a Semi-Markov property into our setup: optimal behaviour of an unemployed individual, and therefore his exit rates, depend not only on whether this individual is unemployed (the current state of the worker) but also on how long has he been unemployed. While this Semi-Markov aspect has been known for a while, it has not been fully exploited so far in the search literature. Using results from the applied mathematics literature (Kulkarni, 1995), we obtain analytic expressions for individual employment probabilities contingent on current employment status and spell duration. They allow us to compute the expected aggregate unemployment rate. Given this link from optimal individual behaviour to aggregate outcomes, we can then analyze the distribution and efficiency effects of changes in level and length of unemployment benefits.

We estimate the model with the data from a German labour market. We ask ourselves what are the welfare consequences of the recent unemployment benefit reforms that have reduced both the magnitude of unemployment assistance benefits and the duration of entitlement to unemployment insurance benefits. We find that the reforms have indeed contributed to the reduction of the unemployment rate. However, our preliminary results show that unemployed and employed workers unambiguously lose. Firms may gain or lose, depending on the strength of the reform. However, in our particular case a loss is much more likely. Our preliminary conclusion therefore points towards a net loss of the social welfare. Returning to the discussion in the beginning of this Introduction, our message is that weakening institutions has a clear limit. Beyond this limit the fight against unemployment should better be continued by other means.

Finally, we note that our model may be easily applied for evaluation of the reforms not only aimed at restructuring a welfare state, but also aimed at setting up a welfare state. This should be largely applicable to any successfully performing developing economy. All the arguments carry over completely, with the only difference that the source of a welfare loss in this case would be a slowdown of production growth due to creating or strengthening the benefit system.

The paper is structured as follows. Section 2 develops the theoretical model. Section 4 deals with the structural econometric model. Section 5 contains preliminary discussion of the effects of benefit reforms.

2 The Model

The model is set in continuous time and concentrates on steady states.

2.1 Production, employment and labour income

The economy has a work force of exogenous constant size N. Employment is endogenous and is given by L. The number of unemployed amounts to $N - L$. Firms produce under perfect competition on the goods market and each worker-firm pair produces constant output A. A worker-firm pair can be separated for exogenous reasons. This occurs according to a homogenous Poisson process with a constant arrival rate λ. There is no search on the job.

Unemployed workers permanently search for a job. Arrival of job offers is modelled by a non-homogenous Poisson process with arrival rate $\mu(., t)$. This rate will also be called the job-finding rate or exit rate (into employment). We allow this rate to depend on effort $\phi(s(t))$ an individual exerts to find a job. Effort today in t depends on the length $s(t)$ of time an individual has spent in unemployment since his last job has been lost. Assuming that unemployment spell starts in t_0, the duration of unemployment spell is $s(t) = t - t_0$. In $t_0 + \bar{s}$, the individual becomes a long-term unemployed worker. Figure 1 displays the event history.

Figure 1: Timing of events

In addition to individual effort, the exit rate will also depend on aggregate labour market conditions captured by labour market tightness θ and on the complementary effect of the rest of relevant exogenous variables. We assume that effort and tightness are multiplicative: no effort implies permanent unemployment and no vacancies implies that any effort is in vain. Complementary effect reflects the aggregate influence of the factors unrelated to the unemployment benefit system reviewed below. From the empirical perspective this is a pure duration-dependence effect.[1] Hence, in full, the exit rate reads $\mu(\phi(s(t))\theta, s)$.

Unemployment benefits depend on the length of the unemployment spell. As long as workers have a spell shorter than $\bar{s}$, they receive unemployment insurance (UI) benefits b_1. Afterwards, when we will call them long-term unemployed, they receive unemployment assistance (UA) benefits b_2, such that $b_2 < b_1$. So, long-term unemployed are those whose duration of unemployment is exceeds the length of entitlement to UI benefits.

$$b(s) = \begin{cases} b_1 & 0 \leq s \leq \bar{s} \\ b_2 & \bar{s} < s \end{cases} \tag{1}$$

Unemployment benefits are financed by a tax rate $\varkappa$ on gross wages such that the net wage is $w = (1 - \varkappa) w^{gross}$. The budget constraint of the government therefore reads

$$\frac{\varkappa}{1 - \varkappa} wL = b_1 U_{short} + b_2 U_{long} \tag{2}$$

where w is the net wage rate. The government adjusts the wage tax $\varkappa$ such that (2) holds at each point in time.

The wage is assumed to be endogenous and is determined by individual bargaining.

2.2 Optimal behaviour

Households

Households live infinitely and consume their entire current income.

Employed agents do not search. So, present value from having a job $V(w)$ is just a sum of the instantaneous utility $u(w,0)$, where 0 stands for zero search effort while working, and the expected capital loss from employer-worker separation. Whenever worker loses the job, he enters the unemployment benefit system by obtaining UI payments b_1 for the full length of $\bar{s}$. Hence, the value of being unemployed when just having lost the job is given by $V(b_1,0)$ where 0 stands for a spell of length zero. Since separations occur at exogenous rate λ, we get the following Bellman equation for employed workers

$$\rho V(w) = u(w,0) + \lambda\left[V(b_1,0) - V(w)\right]. \tag{3}$$

The value $V(w)$ is constant in steady state as the wage is constant.

The Bellman equation for an unemployed worker reads

$$\rho V(b(s),s) = \max_{\phi(s)}\{u(b(s),\phi(s)) + dV(b(s),s)/ds$$
$$+\mu(\phi(s)\theta,s)\left[V(w) - V(b(s),s)\right]\}. \tag{4}$$

The instantaneous utility flow of being unemployed, $\rho V(b(s),s)$, is given by three components. First, the instantaneous utility resulting from consumption of $b(s)$ and effort $\phi(s)$. Second, deterministic changes of $V(b(s),s)$ as the value of being unemployed changes over time (for short-term unemployed). Third, stochastic changes that occur with job-finding rate $\mu(\phi(s)\theta,s)$. When a job is found, an unemployed gains the difference between the value of being employed $V(w)$ and $V(b(s),s)$.

The first order condition of (4) with respect to $\phi(s)$ is given by

$$u_{\phi(s)}(b(s),\phi(s)) + \mu_{\phi(s)}(\phi(s)\theta,s)\left[V(w) - V(b(s),s)\right] = 0, \tag{5}$$

where subscripts denote (partial) derivatives. It states that the expected utility loss resulting from increasing search effort must equal expected utility gains due to a higher search effort. Substitution of (5) into (4) leads to a Bellman equation for an unemployed worker

$$\rho V(b(s),s) = u(b(s),\phi(s)) + \frac{dV(b(s),s)}{ds} - \mu(\phi(s)\theta,s)\frac{u_{\phi(s)}(b(s),\phi(s))}{\mu_{\phi(s)}(\phi(s)\theta,s)} \tag{6}$$

given optimally chosen control variable $\phi(s)$.

We require that the value of unemployment an instant before becoming a long-term unemployed is identical to being long-term unemployed at $\bar{s}$, i.e.

$$V(b_1, \bar{s}) = V(b_2, \bar{s}). \tag{7}$$

Equations (3) and (6)-(7) completely describe the households problem.

Firms

The value of a job is given by instantaneous profits $A - w/(1-\varkappa)$ less the expected capital loss due to match separation,

$$rJ = A - w/(1-\varkappa) - \lambda J. \tag{8}$$

Given that individual arrival rates are a function of the individual unemployment spell, the average arrival rate for workers is just the mean over individual arrival rates, given the distribution of the unemployment spell $f(s)$ to be derived below,

$$\bar{\mu} = \int_0^\infty \mu(s) f(s) \, ds. \tag{9}$$

As a consequence, the vacancy-filling rate is $\xi(t) \equiv \theta^{-1}\bar{\mu}$. The value of holding a vacancy is $rJ_0 = -\gamma + \dot{J}_0 + \xi(t)[J - J_0]$, where γ stands for cost of vacancy. With free entry, we have $J_0 = 0$, i.e.

$$J = \gamma \frac{\theta}{\bar{\mu}}. \tag{10}$$

Equations (8) and (10) completely describe the firms problem.

Wage setting

We let wages be determined by the individual worker-firm bargaining process. We assume that the outcome of the bargaining process is such that workers receive a share β of the total surplus of a successful match

$$V(w) - V(b_1, 0) = \beta \left[J\left(\frac{w}{1-\varkappa}\right) - J_0 + V(w) - V(b_1, 0) \right].$$

The total surplus depends on the gain for the firm and the gain for the worker. The latter depends crucially on the outside option of the worker. We consider here the simple case where all workers (even if only working for an instant) are entitled to full unemployment benefits, i.e. b_1 over the full length $\bar{s}$ and b_2 only for $s > \bar{s}$. This implies that all workers have the same wage, independently of their employment history.

When we then follow the usual steps (Pissarides, 1985), we end up with a wage equation that reads

$$(1-\beta)\,u(w)+\beta\frac{w}{1-\varkappa}=(1-\beta)\,u(b_1,\phi(0))+\beta\,[A+\theta\gamma]\,. \tag{11}$$

The left hand side corresponds to what in models with risk-neutrality and without taxation is simply the wage rate. With $u(w)=w$ and $\varkappa=0$ we would end up with w on the left. The tax rate appears as the term $w/(1-\varkappa)$ results from the instantaneous profit of a firm (8) which needs to pay a gross wage of $w/(1-\varkappa)$. The right hand side is a simple generalization of the standard wage equation of Pissarides (1985) for the case of risk-averse agents and disutility of search effort. Impact of the production side is unchanged compared to the standard equation.

3 Equilibrium

3.1 Definition of equilibrium

The model concentrates on steady states. Below we define the steady state equilibrium.

Definition 1 Steady state equilibrium is a couple $\{w, f(s)\}$, where is w the wage that maximizes match surplus given optimal solution of households and firms problems and $f(s)$ is the endogenous distribution of unemployment duration which satisfies optimal solution of households and firms problems given w.

For a constant and unique value of the output of employer-worker pair A, from (11) follows that the steady state is characterized by unique and constant wage and value of employment. Constant value of employment along with fixed entitlement length $\bar{s}$ consequently leads to a stationary steady state distribution of unemployment duration $f(s)$.

3.2 Functional forms

In order to estimate the model and perform comparative statics analysis we need to make assumptions about functional forms. Let the instantaneous utility function of unemployed worker take the form

$$u(b(s),\phi(s))=\frac{b(s)^{1-\sigma}}{1-\sigma}-\phi(s)\,. \tag{12}$$

and the arrival rate of jobs $\mu(\phi(s)\,\theta,s)$ be

$$\mu(\phi(s)\,\theta,s)=\eta(s)\,[\phi(s)\,\theta]^{\alpha}\,, \tag{13}$$

where $\eta(s)$ is a function that reflects the influence of all exogenous factors other then the design of the benefit system. In general, one can suggest that

$$\eta(s) = \eta_1 G(s) + \eta_2 \tag{14}$$

where $G(s)$ is some continuous of s. Such specification can incorporate a number of interpretations of the complementary effect of exogenous factors. For instance, requiring $\partial G(s)/\partial s < 0$ and $\lim_{s\to\infty} G(s) = 0$, complementary effect $\eta(s)$ can be interpreted as depreciation of search productivity over the duration of unemployment. Of course, one need not necessarily assume that $G(s)$ is a decreasing function. However, for studying long-term unemployment this is a plausible assumption, as typical hazard functions in such a case tend to decrease when s is sufficiently high (see Section 4, p.15 for discussion and justification). Choice of a function form for $G(s)$ is a purely empirical issue.

3.3 Steady state solution

Households problem

Given the above functional forms for the utility function and the exit rate, first-order condition (5) for the optimal search effort implies

$$\phi(s) = \left\{\alpha\eta(s)\theta^{\alpha}\left[V(w) - V(b(s), s)\right]\right\}^{\frac{1}{1-\alpha}}, \tag{15}$$

which holds for both short- and long-term unemployed. With this result, Bellman equation for the unemployed (6) can be is expressed as a differential equation in s

$$\dot{V}(b(s), s) = \rho V(b(s), s) - \frac{b(s)^{1-\sigma}}{1-\sigma} + \frac{\alpha-1}{\alpha}\left[\alpha\eta(s)\theta^{\alpha}\right]^{\frac{1}{1-\alpha}}\left[V(w) - V(b(s), s)\right]^{\frac{1}{1-\alpha}}, \tag{16}$$

which is again valid for both short- and long-term unemployed. When going from being short- to long-term unemployed, the transition condition

$$V(b_1, \bar{s}) = V(b_2, \bar{s})$$

must hold. Finally, assuming $\lim_{s\to\infty} G(s) = 0$, value of unemployment becomes constant as $\lim_{s\to\infty} \eta(s) = \eta_2$. This provides the terminal condition for (16), which reads

$$\rho V(b_2) = \frac{b(s)^{1-\sigma}}{1-\sigma} - \frac{\alpha-1}{\alpha}\left[\alpha\eta_2\theta^{\alpha}\right]^{\frac{1}{1-\alpha}}\left[V(w) - V(b_2)\right]^{\frac{1}{1-\alpha}} \tag{17}$$

and has a unique solution for $V(b_2)$, given any $V(w)$ and θ. Bellman equation for employed worker (3) determines the value of a job

$$V(w)=\frac{1}{\rho+\lambda}\left(\frac{w^{1-\sigma}}{1-\sigma}+\lambda V(b_1,0)\right) \tag{18}$$

for any given wage w.

The solution of firms problem provides us with w and θ.[2]

Firms problem and wage setting

Firms problem is much easier as there is no time-dependence in it. The number of vacancies, represented by labour market tightness θ, follows from (8) and (10) giving us

$$\frac{A-\frac{w}{1-\varkappa}}{\rho+\lambda}=\gamma\frac{\theta}{\bar{\mu}}. \tag{19}$$

The wage w comes from bargaining and is determined by

$$(1-\beta)\,u(w)+\beta\frac{w}{1-\varkappa}=(1-\beta)\,u(b_1,\phi(0))+\beta\left[A+\theta\gamma\right], \tag{20}$$

where $\phi(0)$ is the optimal search effort at the instant of entry into unemployment, which is a part of the solution of households problem.

Equilibrium distribution of unemployment duration

The final step in closing the model is the determination of the equilibrium distribution of unemployment duration.

As defined in (9), steady state rate of inflow into employment $\bar{\mu}$ is the expected individual exit rate out of unemployment, where expectation is taken over the distribution of unemployment duration $f(s)$. Using a well-known dependence between exit rate and probability distribution function (see e.g. Ross, 1996, p.78-80) for a non-homogeneous Poisson process with arrival rate $\mu(s)$, the probability density of waiting time s in a state since the moment of entry is given by

$$f(s)=\mu(s)\exp\left[-\int_0^s\mu(u)\,du\right].$$

Denoting the optimal search effort of short-term unemployed by $\phi_1(s)$ optimal search effort of long-term unemployed by $\phi_2(s)$, both follow from (15), in our case we get

$$f(s)=\begin{cases}\mu(\phi_1(s)\,\theta)\,e^{-\int_0^s\mu(\phi_1(u)\theta)du} & \text{for } s\leq\bar{s}\\ \frac{\exp\left\{-\int_0^{\bar{s}}\mu(\phi_1(u)\theta)du\right\}}{\exp\left\{-\int_0^{\bar{s}}\mu(\phi_2(u)\theta)du\right\}}\mu(\phi_2(s)\,\theta)\,e^{-\int_0^s\mu(\phi_2(u)\theta)du} & \text{for } s>\bar{s}\end{cases}. \tag{21}$$

The second line in (21) is the probability of surviving $\bar{s}$ with a high level of benefit payments times the density of unemployment duration conditional on the expiration of entitlement, i.e. on $s > \bar{s}$, and transition to a lower level of benefit payments.

Applying (21) to form $\bar{\mu}$ and substituting it into (19) we close the model.

Finally, with the density in (21) we can also compute the number of short-term and long-term unemployed simply by $U_{short} = N \int_0^{\bar{s}} f(s)\, ds$ and $U_{long} = N - L - U_{short}$. This allows us to compute the tax rate $\varkappa$ which makes the government budget constraint hold,

$$\varkappa = \frac{\frac{b_1 U_{short} + b_2 U_{long}}{wL}}{1 + \frac{b_1 U_{short} + b_2 U_{long}}{wL}}. \tag{22}$$

3.4 Individual (un)employment probabilities

In models with constant job-finding and separation rates, the unemployment rate can easily be derived by assuming that a law of large numbers holds. Employment dynamics can then be described by $\dot{L} = \mu [N - L] - sL$ which allows to compute unemployment rates. With spell-dependent effort, individual arrival rates $\mu(.)$ are heterogeneous and employment dynamics need to be derived using techniques from the literature on Semi-Markov processes (e.g. Kulkarni, 1995).

The generalization of Semi-Markov processes compared to memoryless Markov chains consists in allowing transition rates from one state to another, i.e. here the arrival rate of a job when unemployed, to depend on the time an individual has spent in a state, i.e. here the time s the individual has been unemployed.

We start by looking at individual employment probabilities. Let $p_{ij}(\tau, s(t))$ describe the probability that an individual which is in state i (either e for employed or u for unemployed) today in t is in state j at some future point in time τ when his current unemployment spell is $s(t)$. We can show that these expressions read, taking already into account that the separation rate λ remains constant,

$$p_{uu}(\tau, 0) = e^{-\int_t^{\tau} \mu(s(y))dy} + \int_t^{\tau} e^{-\int_t^{v} \mu(s(y))dy} \mu(s(v))\, p_{eu}(\tau - v)\, dv, \tag{23a}$$

$$p_{eu}(\tau) = \int_t^{\tau} e^{-\lambda[v-t]} \lambda p_{uu}(\tau - v, 0)\, dv. \tag{23b}$$

Expressions for other transitions are given by $p_{ue}(\tau) = 1 - p_{uu}(\tau)$ and $p_{ee}(\tau) = 1 - p_{eu}(\tau)$.

These equations have a straightforward intuitive meaning. Consider first the case of τ being not very far in the future. Then all integrals (for $\tau = t$) are zero and the probability of being unemployed in τ is 1 when being unemployed in t (23a) and zero when having a job (23b) in t. For a $\tau > t$, the part $e^{-\int_t^\tau \mu(s(y))dy}$ in (23a) gives the probability of remaining in unemployment for the entire period from t to τ. An individual unemployed today can also be unemployed in the future, however, if he remains unemployed from t to v (the probability of which is $e^{-\int_t^v \mu(s(y))dy}$ in the integral), loses the job in v (which requires multiplication with the exit rate $\mu(s(v))$) and then moves from employment to unemployment again over the remaining interval $\tau - v$ (for which the probability is $p_{eu}(\tau - v)$). As this path is possible for any v between t and τ, the densities for these paths are integrated up. The sum of the probability of remaining unemployed all of the time and of finding a job at some v but being unemployed again at τ gives then the overall probability $p_{uu}(\tau, 0)$ of having no job in τ when having no job in t. Note that there can be an arbitrary number of transitions in and out of employment between v and τ.

The interpretation for (23b) is similar. The probability of remaining employed from t to v is simpler, $e^{-\lambda[v-t]}$, as the separation rate λ is constant. As we can see, these equations are interdependent: The equation for $p_{uu}(\tau)$ depends on $p_{eu}(\tau - v)$ and the equation for $p_{eu}(\tau)$ in turn depends on $p_{uu}(\tau - v)$. Formally speaking, these equations are integral equations, sometimes called Volterra equations of the first type (23b) and of the second type (23a). Integral equations can sometimes be transformed into differential equations and then be treated numerically by standard software. In our case, however, no transformation into differential equations is known and we need to solve them numerically.

We have computed the probability of being unemployed in τ when being unemployed in t only for individuals that just became unemployed in t, i.e. who have a spell of length $s(t) = 0$. What we will need in what follows, however, is an expression for $p_{uu}(\tau, s(t))$, i.e. for individuals that have an arbitrary spell of unemployment. Luckily, given the results from (23a and b), this probability is straightforwardly given by

$$p_{uu}(\tau, s(t)) = e^{-\int_t^\tau \mu(s(y))dy} + \int_t^\tau e^{-\int_t^v \mu(s(y))dy} \mu(s(v))\, p_{eu}(\tau - v)\, dv. \tag{23c}$$

An unemployed with spell $s(t)$ in t has different exit rates $\mu(s(y))$ which, however, are known from our analysis of optimal behaviour at the individual level. So, only the integrals in (23c) are different; the probabilities $p_{eu}(\tau - v)$ can be taken from the solution of (23a and b).

3.5 Aggregate equilibrium unemployment

Given our finding in (23) on $p_{eu}(\tau)$ and $p_{uu}(\tau, s(t))$, we can now compute our expected number of unemployed for any distribution of spell $F(s)$,

$$E_t[N-L_\tau] = [N-L_t]\int_0^\infty p_{uu}(\tau, s(t))\, dF(s(t)) + p_{eu}(\tau) L_t. \tag{24}$$

Starting at the end of this equation, given L_t employed workers in t, the expected number of unemployed for some future point in time τ out of the group of the currently employed is given by $p_{eu}(\tau) L_t$. Again, one should keep in mind that the probability $p_{eu}(\tau)$ allows for an arbitrary number of switches between employment and unemployment between t and τ, i.e. it takes the permanent turnover into account.

For the unemployed, we compute the mean over all probabilities of being unemployed in the future when unemployed today by integrating over $p_{uu}(\tau, s(t))$ given the current distribution $dF(s(t))$. Multiplying by the number of unemployed today, $N-L_t$, gives the expected number of unemployed out of this pool. Summing these two expected quantities gives the expected number of unemployed at some future point τ.

The expected unemployment rate for τ is simply this expression divided by N. When we focus on a steady state, we let τ approach infinity. If we are willing to assume a law of large numbers where the share of the population being unemployed equals the average individual probability of being unemployed, we could remove the expectations operator and consider the expected unemployment rate to represent the realized unemployment rate, $u_\tau = E_t[1 - L_\tau/N]$.

4 Estimation

In this section we consider various specifications of the structural econometric model and discuss the estimation results obtained from the best specification.

4.1 Exit rates

Solution of the theoretical model developed in Section 2 provides us with the endogenous distribution of unemployment duration characterized by the probability density (21). This density is the key element in the construction of the likelihood function. Considering (21), all we need to know from the theory is the exit rates before ($s \leq \bar{s}$) and after ($s > \bar{s}$) expiration of entitlement to UI benefits. Below we consider these exit rates.

One insightful special case

Let us for the moment assume that the only source of time-dependence of the exit rate out of unemployment is the difference between benefit levels before and after expiration of entitlement. So the effect of all other exogenous factors is time-constant and $G(s) = 0$ for any s, leading to $\eta(s) = \eta_2$, as (14) suggests. Furthermore, let us assume that α in (13) is equal to 1/2. Under these two assumptions Bellman equation (16) that describes the value of unemployment before the expiration of entitlement becomes a quadratic differential equation with time-invariant coefficients

$$\dot{V}(b_1, s) = -\frac{\eta_2^2\theta}{4}V(b_1, s)^2 + \left(\rho + \frac{\eta_2^2\theta}{2}V(w)\right)V(b_1, s) - \left[\frac{\eta_2^2\theta}{4}V(w)^2 + \frac{b_1^{1-\sigma}}{1-\sigma}\right] \tag{25}$$

and Bellman equation (17) for the value of unemployment after the expiration of entitlement reduces to a simple quadratic equation

$$\frac{\eta_2^2\theta}{4}V(b_2)^2 - \left(\rho + \frac{\eta_2^2\theta}{2}V(w)\right)V(b_2) + \left[\frac{\eta_2^2\theta}{4}V(w)^2 + \frac{b_2^{1-\sigma}}{1-\sigma}\right] = 0 \tag{26}$$

with no time dependence. Terminal condition for (25) remains $V(b_1, \bar{s}) = V(b_2)$, with the only difference that from $s = \bar{s}$ onward a worker finds himself in an entirely stationary environment. Using equations 1.2.2-24 and 2.1.2-11 from Polyanin and Zaitsev (2003, p.84 and p.215 respectively) one can discover that (25) has an analytical solution for $V(b_1, s)$. Substituting this solution into (15) and applying (13), for a given value of employment $V(w)$ we get analytical expressions for the exit rates before and after $\bar{s}$

$$\mu_1(\phi_1(s)\theta) = \eta_2\sqrt{B_1}\frac{1 + \frac{B-1}{B+1}e^{\eta_2(s-\bar{s})\sqrt{B_1}}}{1 - \frac{B-1}{B+1}e^{\eta_2(s-\bar{s})\sqrt{B_1}}} - \frac{\rho}{\sqrt{\theta}}, \qquad \text{for } s \le \bar{s} \tag{27a}$$

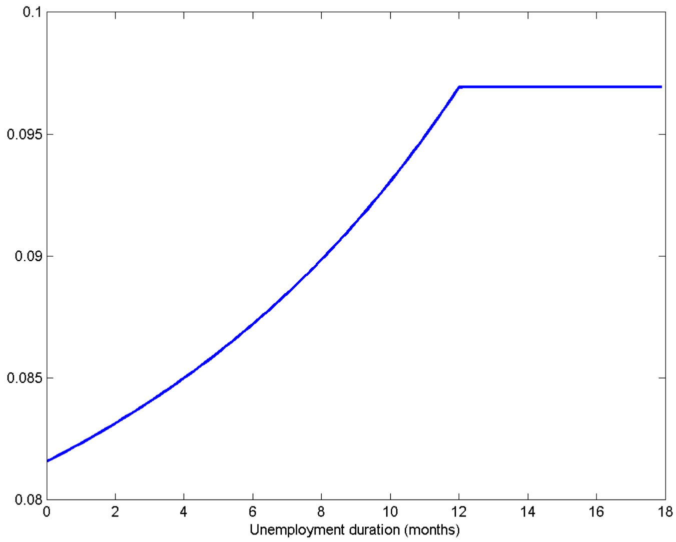
$$\mu_2(\theta) = \eta_2\sqrt{B_2} - \frac{\rho}{\sqrt{\theta}}, \qquad \text{for } s > \bar{s} \tag{27b}$$

where $B_j \equiv \rho V(w) - b_j^{1-\sigma}/(1-\sigma) + \theta^{-1}(\rho/\eta_2)^2$, $j = 1, 2$, and $B \equiv \sqrt{B_2/B_1}$.

Taking first derivative of (27a) with respect to s one can easily see that $\partial\mu_1/\partial s > 0$ for $s \le \bar{s}$, since $b_2 < b_1$. Of course, for μ_2, $\partial\mu_2/\partial s = 0$. Thus, our special case implies that once nonstationarity of unemployment benefit system is the only source of time-dependence, optimal choice of search effort leads to a steadily increasing hazard function. This increase starts from the very entry into unemployment and continues up until the expiration of entitlement period. Once the entitlement period is over, search becomes stationary again and the reemployment risk is constant.

For typical parameter values and values of wage, unemployment benefits and entitlement length, hazard function implied by our special case will look as follows

Figure 2: Pure benefit effect

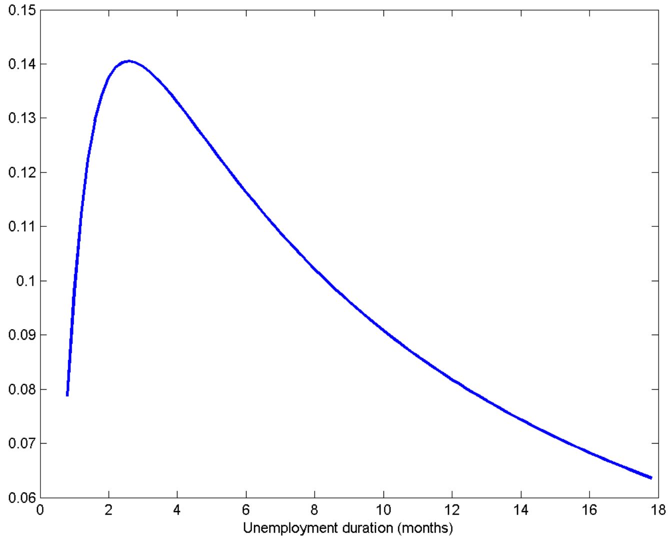


This is nothing but the classical benefit effect that was first discussed in a one-sided search model of Mortensen (1977). Identical hazard rates with increasig-constant profile also appear in the later literature (again, only in the one-sided setting; see van den Berg, 1990, or Fredriksson and Holmlund, 2006, for an overview).

Nevertheless, the assumption of the hazard rate depending on the design of the benefit system exclusively can be easily challenged. Taking the data we later use for the estimation of our structural model, let us, first, fit the unconditional generalized gamma model for the duration of unemployment. This model is characterized by a flexible parametric hazard function that can be both monotone and nonmonotone. Figure 3 below plots the predicted hazard function.

From Figure 3 we see that after the first four months hazard function has a clear downward slope. Thus, along with the benefit effect, that drives the hazard up, there must exist a complementary non-benefit effect that pushes it down. This urges for generalization of the simple model. Returning to the restrictions made above, relaxing the assumption $\alpha = 1/2$ and letting α vary free cannot bring the desired improvement, as this will only change the steepness of the hazard function before the expiration of entitlement. After the expiration, however, the exit rate will still remain flat. Therefore the assumption of time-invariance of the complementary effect, i.e. $G(0) = 0$, should be addressed. In particular, we have to suggest that $\partial G(s)/\partial s < 0$ holds at least for sufficiently high s. This will insure that the exit rate goes down once entitlement to UI benefits expires.

Figure 3: Generalized gamma model



The argument above also provides the empirical justification for both the terminal condition $\lim_{s\to\infty} G(s) = 0$, which we use in the theoretical formulation (see p.9), and for the inclusion of the non-benefit effect $\eta(s)$ into the theoretical model itself.

Generalization

Following the above discussion we let $\eta(s)$ be a continuous decreasing function of s. Recalling the definition of $\eta(s)$ in (14) a natural candidate for $G(s)$ would be a survivor function of any positive-valued continuous random variable. However, our choice of specification for $G(s)$ is restricted by both the danger of overparameterizing the non-benefit effect and the identifiability considerations.

Once we allow for time-dependent non-benefit effect, even for $\alpha = 1/2$ Bellman equations (16) and (17) for the value of unemployment before and after the expiration of entitlement period will not have analytical solutions any longer. Consequently, we will not be able to get analytical solutions for the hazard functions. Using (15) and (13) one can show that for a given value of employment $V(w)$ the exit rate beyond the expiration of entitlement solves

$$\begin{aligned}\dot{\mu}_2(\phi_2(s)\theta, s) = {} & \frac{\sqrt{\theta}}{2}[\mu(\phi_2(s)\theta, s)]^2 \\ & + \left(2\frac{\partial\eta(s)/\partial s}{\eta(s)} + \rho\right)\mu(\phi_2(s)\theta, s) - \frac{[\eta(s)]^2\sqrt{\theta}}{2}\left[\rho V(w) - \frac{b_2^{1-\sigma}}{1-\sigma}\right] \qquad (28)\end{aligned}$$

with the terminal condition

$$\mu\left(\phi\left(s\right)\theta,s\right)=\eta_{2}\sqrt{\rho V\left(w\right)-\frac{b_{2}^{1-\sigma}}{1-\sigma}+\frac{\rho^{2}}{\eta_{2}^{2}\theta}}-\frac{\rho}{\sqrt{\theta}}, \tag{29}$$

where the latter obtains from $\lim\limits_{s\to\infty}\eta(s)=\eta_2$. The exit rate before the expiration of entitlement to UI benefits solves nearly identical differential equation

$$\begin{aligned}\dot{\mu}_{1}\left(\phi_{1}\left(s\right)\theta,s\right)=&\frac{\sqrt{\theta}}{2}\left[\mu_{1}\left(\phi_{1}\left(s\right)\theta,s\right)\right]^{2}\\&+\left(2\frac{\partial\eta\left(s\right)/\partial s}{\eta\left(s\right)}+\rho\right)\mu_{1}\left(\phi_{1}\left(s\right)\theta,s\right)-\frac{\left[\eta\left(s\right)\right]^{2}\sqrt{\theta}}{2}\left[\rho V\left(w\right)-\frac{b_{1}^{1-\sigma}}{1-\sigma}\right]\end{aligned} \tag{30}$$

given the terminal condition

$$\mu_{1}\left(\phi_{1}\left(\bar{s}\right)\theta,\bar{s}\,\right)=\mu_{2}\left(\phi_{2}\left(\bar{s}\right)\theta,\bar{s}\right). \tag{31}$$

The only difference between (28) and (30) is the level of instantaneous benefit payments (b_2 and b_1 respectively).

In Appendix we show that for typical parameter values and particular choices of $G\left(s\right)$, hazard functions μ_1 and μ_2 that solve (28) and (30) are capable of exhibiting various patterns of non-monotone behaviour.

4.2 Likelihood function

Data and sampling

The model is estimated using the data from the German Socio-Economic Panel (GSOEP), which is a panel survey of individuals conducted on the annual basis. Since GSOEP data contain no information about firms and linked employer-employee data are unavailable to us, when estimating the model we must treat wages as given. Implicitly this means that observed wages are viewed as endogenous outcomes of some general wage setting process, which may not necessarily be the individual worker-firm Nash bargaining. However, being consistent with a general wage setting process parameters of the model estimated with workers' side only will also be consistent with wage bargaining as described in Section 2. So, with the parameters estimated from this type of data and any consistent estimates of the average firm productivity A and cost of vacancy γ, we will still be able to evaluate the effects of benefits reforms correctly.[3]

The data we use are sampled as a flow of entrants to employment and unemployment at each month of year 1997. The choice of the year of sampling is determined by the fact that no changes to either benefit level or entitlement length were made between the

1st of January 1997 and the 1st of January 2005, when Hartz IV reform came into power. With December 2003 being the latest month of our observation period we end up with a sample that describes a stationary entitlement-benefit environment and provides a fairly reliable information on long-term unemployment (only 5.5% of unemployment durations in our sample are right-censored).

Finally, it is important to notice that GSOEP data do not contain information on the length of entitlement to UI benefits. There exist, however, strict and relatively simple rules that allow computing the length of entitlement once we know the length of previous job durations and the age of an individual. For this reason, for every person entering unemployment we also have to retrieve his/her previous job history. In addition to that, previous job history provides us with the record of the latest wage earned.

Units of measurement are months for the duration data and German Marks for the wage data. More information about the data can be found in Appendix.

Individual contributions

Treating w as given, value of employment $V(w)$ can be computed as described in Endnote 1 to this article (see also p.10). Once value of employment is known, exit rates immediately follow. Given that the data are sampled as a flow, exit rates contain the entire information relevant for the construction of the likelihood function (see, e.g. Lancaster, 1990).

Our data contain three types of unemployed individuals:

a) Individuals who enter unemployment with the right to claim UI benefits and exit unemployment before the expiration of entitlement period

b) Individuals who enter unemployment with the right to claim UI benefits, fail to find a job before entitlement expires, transit to a lower UA benefits and exit unemployment (or not) only after the expiration entitlement

c) Individuals who do not have the right to claim UI benefits and enter unemployment receiving lower UA benefits from the very beginning (if at all)

Let $\boldsymbol{\xi}$ denote the vector of parameters to estimate, which are $\{\lambda, \sigma, \eta_1, \eta_2, \boldsymbol{\delta}\}$, where $\boldsymbol{\delta}$ stands for a parameter vector that determines $G(s)$.[4] Furthermore let l_i define the length of previous/current individual job. Keeping in mind that job loss is governed by a homogeneous Poisson process with rate λ, which implies exponential distribution of the length of job spell, we get the following log-contributions for the above types of individuals

$$a): \quad \ln \ell_i(\boldsymbol{\xi}) = d_{j,i} \ln \lambda - \lambda l_i + d_{u,i} \ln \mu_1(s_i; \boldsymbol{\xi}) - \int_0^{s_i} \mu_1(u; \boldsymbol{\xi})\, du \tag{32a}$$

$$b): \quad \ln \ell_i(\boldsymbol{\xi}) = d_{j,i} \ln \lambda - \lambda l_i + \left[\int_0^{\bar{s}_i} \mu_2(u; \boldsymbol{\xi})\, du - \int_0^{\bar{s}_i} \mu_1(u; \boldsymbol{\xi})\, du \right] + d_{u,i} \ln \mu_2(s_i; \boldsymbol{\xi}) \quad (32b)$$

$$c): \quad \ln \ell_i(\boldsymbol{\xi}) = d_{j,i} \ln \lambda - \lambda l_i + d_{u,i} \ln \mu_2(s_i; \boldsymbol{\xi}) - \int_0^{s_i} \mu_2(u; \boldsymbol{\xi})\, du, \quad (32c)$$

where $d_{u,i}$ is an indicator variable such that $d_{u,i} = 1$ if unemployment spell is uncensored and $d_{j,i}$ is an indicator variable such that $d_{j,i} = 1$ if job spell is uncensored. Finally, log-contribution of entrants to employment is simply

$$\ln \ell_i(\boldsymbol{\xi}) = d_{j,i} \ln \lambda - \lambda l_i \quad (32d)$$

The total log-likelihood function is the sum of all individual log-contributions, $i = 1, .., n$.

4.3 Estimation results

First we discuss the specification of non-benefit effect. We start with a fairly general expression for $\eta(s)$, assuming that $G(s)$ has a shape of the survivor function of a Weibull distribution, so that

$$\eta(s) = \eta_1 \exp\left\{-\delta_1 s^{\delta_2}\right\} + \eta_2 \quad (33)$$

In this form $\eta(s)$ goes down relatively slow in the beginning of an unemployment spell, relatively fast in the middle of it and relatively slow again, when s is sufficiently large. Setting $\delta_2 = 1$, which reduces $G(s)$ to the survivor function of an exponential distribution, $\eta(s)$ will go down relatively fast right from the beginning of the unemployment spell and relatively slow thereafter. Finally, with $\delta_1 = 0$ complementary effect ceases to be time-dependent. Despite for the reasonable parameter values and average individual characteristics the most general form of $\eta(s)$ provides the best-looking hazard function (see Appendix), our particular data seem to tell that specification in (33) largely overparameterizes the effect. A medium-size sensitivity analysis shows that without setting $\delta_2 = 1$ and $\eta_1 = \eta_2$ the model yields a flat likelihood profile in the neighborhood of the maximum, either failing to converge ($\delta_2 \neq 1$) or providing unreliable standard errors ($\delta_2 = 1$, $\eta_1 \neq \eta_2$). Therefore our final specification for $\eta(s)$ becomes

$$\eta(s) = \eta(1 + \exp\{-\delta s\}) \quad (34)$$

This leads to two different models: Model 1 ($\delta = 0$), which is a special case with pure benefit effect only, and Model 2 ($\delta \neq 0$), which is a generalization that incorporates both benefit and complementary non-benefit effects.

The estimates of the structural parameters are presented in Table 1.

Table 1: "Estimation results" [a]

	Model 1			Model 2		
Parameter	Coeff.	Std.Err.	p-Value	Coeff.	Std.Err.	p-Value
λ	0.0140	0.0007	0.0000	0.0141	0.0007	0.0000
η	0.0047	0.0023	0.0388	0.0029	0.0009	0.0007
δ				0.0571	0.0175	0.0011
σ	0.1507	0.1369	0.2709	0.1836	0.0817	0.0246
log-Likelihood		- 3187.28			- 3171.32	
LRT	*Test Stat.* $[\chi^2_{(1)}]$:		31.9203	*p-Value*:		0.0000

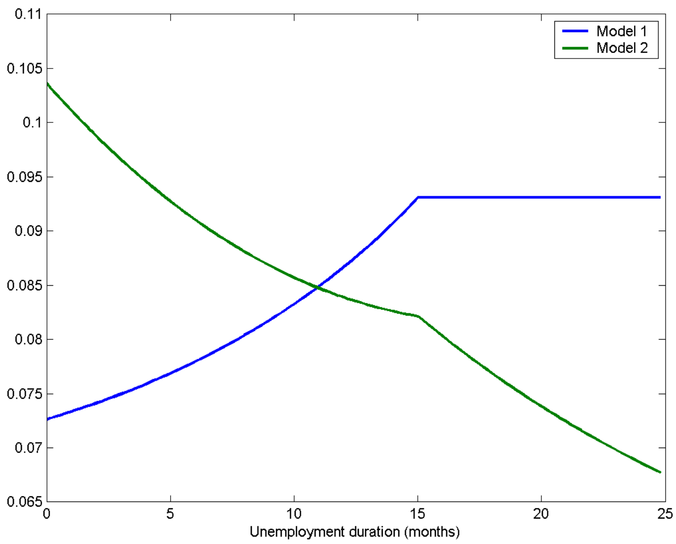
[a] Both models are estimated in Matlab using BFGS method and ode45 solver for differential equations. Convergence to unique maximum obtains from a variety of starting values and maximizer lies in the interior of the parameter space

From Table 1 we see that modelling both benefit effect and the influence of the rest of exogenous variables simultaneously results in a much better fit to the data. Furthermore, the extended specification underlines the significance of risk-aversion, which can be seen from the significance of the estimate of σ.

The power of the extended specification can be further seen from Figure 4 which plots predicted hazard rates for an individual with average characteristics (see Table A1 in the Appendix). From this figure we can see that, contrary to the incentive design of the unemployment benefit system, instantaneous reemployment probability goes down throughout the entire duration of unemployment spell. However, as the expiration of entitlement comes closer and search effort increases more and more rapidly, instantaneous reemployment probability decreases by a much slower rate.

Though the main message this figure sends us is that there exist two significant counterforces that influence the hazard function, one being exclusively unemployment benefit system and another being the complement, unrelated to the design of the system. And our model is capable of separating these two counterforces completely. This makes it an attractive tool for studying any kind of stepwise policy design. An example considered in this paper is a reduction of UA benefits and an entitlement cut, which would be typical for welfare states. However, in the same framework one can easily consider a reciprocal example that introduces some positive level of unemployment benefit payments in the economy, which previously did not have any state support for unemployed workers. This setting would be typical for a number of successfully performing developing countries.[5]

Figure 4: Structural hazard functions



Finally, it is straightforward to conditionalize the model by suggesting $\eta = \exp\{\mathbf{x}_i'\boldsymbol{\beta}\}$. Though in the present application we are interested in the unconditional version only, as all our analysis is done exclusively on the aggregate level.

5 Unemployment benefit reform

In this section we present a preliminary discussion of immediate effects and welfare consequences of Hartz IV reform in Germany.

5.1 Key features

The reform was designed to address two key components of unemployment benefit system: unemployment assistance benefits (b_2) and the duration of entitlement to unemployment insurance payments ($\bar{s}$). According to pre-reform legislation, monthly unemployment assistance payments, in case claimable, were at the level of 57% of the previous net earnings, once an individual had at least one dependent child, and 53% of the previous net earnings for those without children. Hartz IV reform abolishes these replacement rates and introduces a flat benefit level of €345 irrespective of the magnitude of previous earnings and the number of dependent children. Pre-reform lengths of entitlement to UI benefits were varying from 6 to 32 months depending on both the age and the count of UI contribution payments during previous employment. However, irrespective of the history of contribution payments, no person below 45 years of age was able to claim unemployment insurance benefits for a period longer than 12 months. Hartz IV reform abolishes age-differentiation for the calculation of entitlement period and sets the upper cap of 12 months irrespective of the age, given that the length of contribution period is at least 24 months within last seven years.

Our data show that of the two components above UA benefit reduction is relatively more important as 55% become subject to a benefit cut, whereas only 20% are affected by a reduction in the length of entitlement. In addition to that, the loss of those who loose from the benefit cut (€200 on average) is bigger than the gain those who gain (€125 on average).

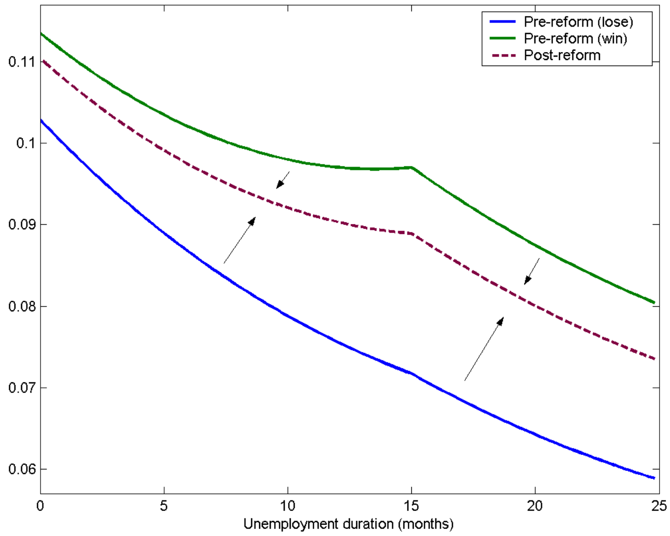
The primary goal of the reform was the reduction of unemployment. Event though, this seems to have largely worked, we ask ourselves what is the social price and what are welfare consequences of this reduction? The analysis below provides first answers.

5.2 Hartz IV reform and reemployment risk

First, let us consider the change in UA payments. As articulated before, this part of the reform was of advantage for those, who were previously receiving too low unemployment assistance benefits. Once the reform came into power value of unemployment for this segment of workers has gone up, implying consequently lower search intensity and lower instantaneous probability of reemployment. The opposite is true for those who have lost from the reform.

Figure 5 helps us looking into the difference between the exit rates of both kinds of individuals. Blue line in this figure corresponds to the exit rate of an individual with average characteristics of a subsample of workers that lose from the benefit cut (i.e., workers whose value of b_2 exceeds €345 by the moment the reform is implemented). Green line reflects the exit rate of the worker with monthly benefits lower than €345. Finally, the dashed purple line shows the post-reform intensity with the uniform level of assistance payments. We see effort increase of those who lose is much higher than the indulgence of those who win.

Figure 5: Hazard functios and UA benefits



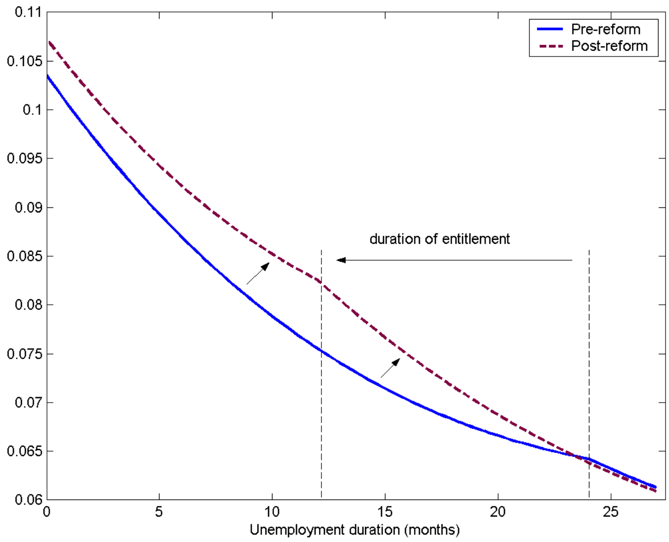
Once reaching the end of the entitlement period, the probability of finding a job at $\bar{s}$ among the "losers" is 25% higher than before the reform, whereas the same probability among the "winners" is only about 10% lower than what used to be earlier. Given that the shares of positively and negatively affected individuals are about the same, this figure implies that, on the net, UA benefit cut has lead to about 10% increase in reemployment risk in the entire economy. Consequently it is logical to expect that the aggregate unemployment rate will go down.

Consider now the reemployment risk induced by the reform of entitlement. There are no winners from this reform. In addition to that, for an average individual subject to the reform (i.e. the one with $\bar{s}$ over 12 months) the cut in entitlement length turns out to be quite substantial, namely from 24 months, on average, to 12 months. Nevertheless, the increase in reemployment risk, associated with this cut is not so large and reaches only about 10% in its peak. Figure 6 on the next page illustrates the difference between pre- and post-reform hazard functions. Keeping in mind that only 1/5 of all unemployed individuals are affected by this policy measure, it is easy to see that on the aggregate level, change in $\bar{s}$ should have weaker influence on the unemployment rate, if compared with change in b_2.

5.3 Welfare implications of Hartz IV reform

The above analysis of reemployment risks induced by the reform has lead us to a conclusion that the aggregate unemployment is likely to go down, once the reform is implemented.

Figure 6: Hazard functions and duration of entitlement



Though we would be more interested in the share of long-term unemployed in total unemployment. To analyze this quantity and, in general, all the welfare effects of the Hartz IV reform we need to conduct a number of comparative statics exercises. We define the pre-reform steady state by the vector of estimated parameters and mean values of the relevant variables observed in the data.[6]

Using the estimation results and the sample average for the wage and assuming that tightness equals to one, under symmetric wage bargaining we can predict the values of average firm productivity A and cost of vacancy γ. These quantities are obtained with the help of (19)-(21), where $\bar{s}$ is, again, taken from the data. Finally, we find it more convenient

Table 2: "Pre-reform steady state"

I. Estimated parameters (**old!**)

λ	η	α	σ
0.02	0.09	0.02	0.7

II. Exogenous parameters

ρ	β	b_1/w	b_2/w	$\bar{s}$
.003	.5	.58	.47	15.8

III. Predicted par.

A	γ
DM 2251.5	4.1

IV. Equilibrium values

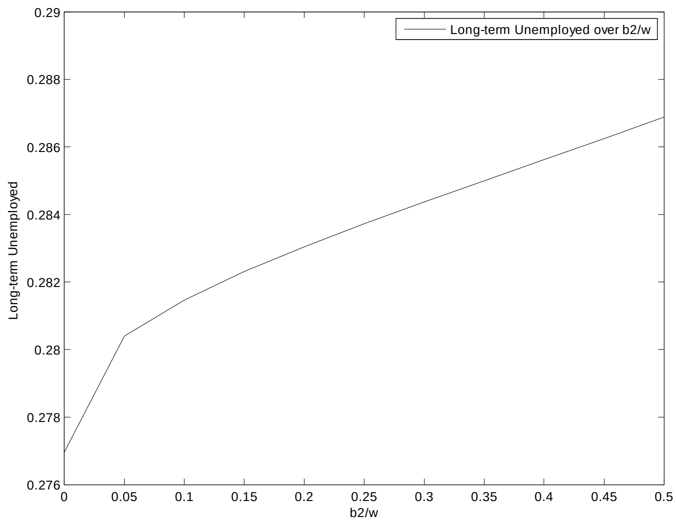
w	θ	u
DM 2250	1	17%

to consider all comparative statics analysis in terms of replacement rate b_2/w rather than in terms of benefit level. Table 2 shows the values that describe pre-reform steady state. The unemployment rate u in Table 2 is the endogenous outcome of $u = \lim_{\tau\to\infty} E_t\left[1 - L_\tau/N\right]$ from (24), given the estimated parameters.

Long-term Unemployment

We now ask, what does the reduction of unemployment assistance benefits imply for the share of long-term unemployed? Figure 7 provides the answer. The x-axis of this figure plots the replacement rate with the pre-reform value of b_2/w being the rightmost point. Hartz IV reform has reduced the average replacement rate, so to trace the effect of the reform in this picture, one needs to start from the rightmost and move towards zero.

Figure 7: Unemployment rate (share of long-term unemployed)



We see that the share of long-term unemployed decreases as the replacement rate goes down. This means that introduction of a flat UA benefit which, on average, is less than pre-reform 50% of the net earnings, means that the share of long-term unemployment is to fall as a result of the reform.

So, the evidence collected from Figures 5-7 lead us toward a conclusion that in terms of both for the aggregate and the long-term unemployment, the Hartz IV reform is most likely a success. But is it really?

One of the strongest challenges Hartz IV reform has been facing starting from the early stages of its development is the danger of the net social welfare loss. To see whether this danger has materialized into the actual loss we need to consider how welfare of the representative agents on both sides of the market was reacting to the reduction of unemployment assistance benefits and entitlement length. Supply side of the market encompasses both employed and unemployed workers. Demand side is a representative firm.

UA benefits and individual welfare

Let us first consider the supply side. Below we plot the value of employment $V(w)$ against the replacement rate.

Figure 8: Welfare of employed worker

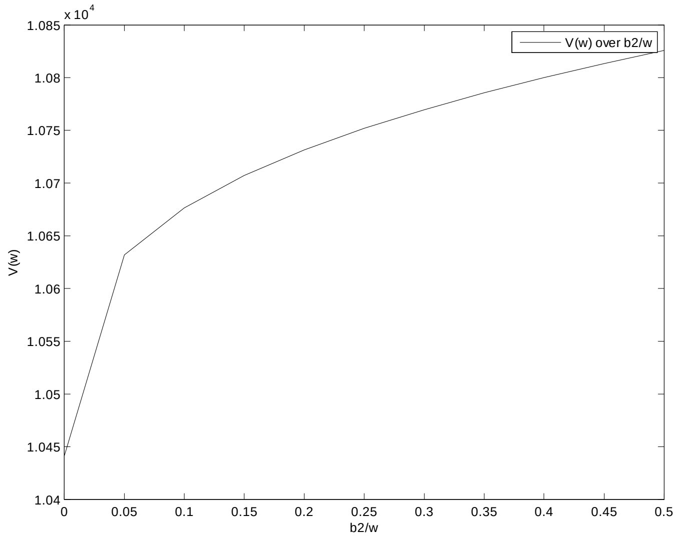
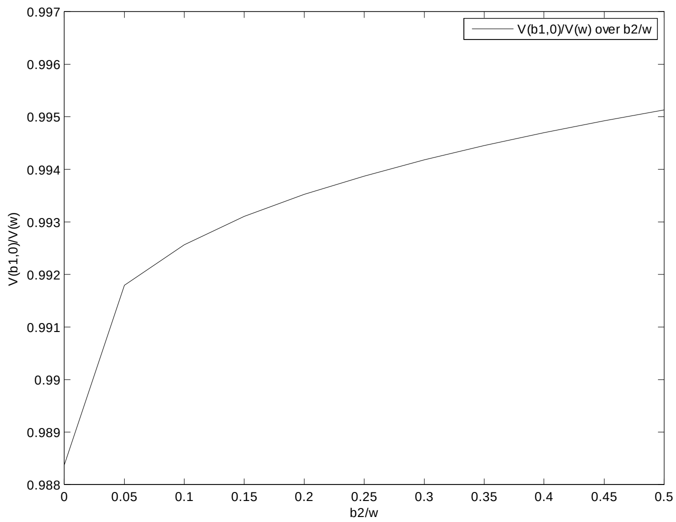


Figure 8 tells us that employed agents unambiguously lose from the reduction of b_2. The key to understanding this result is provided by equation (18), that describes the value of employment, and equation (20) that defines the equilibrium wage. As we can see from (18), the value of a job positively depends on both the wage and the outside option of employment, which is the entry into unemployment with b_1-b_2 step system. Now, reduction of assistance payments definitely reduces the value of the outside option, so an least on this end $V(w)$ must go down. Considering the wage, from (20) one can state that wage decreases in search effort, as $\partial u(b, \phi) / \partial \phi < 0$, and increases in market tightness. Thus, the reform influences the wage in two ways: $^{1)}$ benefit reduction enhances search intensity, so the wage goes down [effort effect], $^{2)}$ benefit reduction decreases unemployment, so the tightness goes down and the wage goes up [employment effect]. Once effort effect dominates, equilibrium wage decreases. Otherwise, wage must fall. This relatively simple response pattern provides us with all information relevant for understanding the behaviour of the value of the job. As we have stated before, the value of an outside option has gone down. In addition to that, either the effort effect was stronger than employment effect, which drove $V(w)$ further down, or positive net effect of the equilibrium wage was insufficient to offset the too sharp decrease in the value of the outside option. In any of the two cases the value of the job falls and employed workers loose from the reduction of unemployment assistance payments. Exactly this is shown in Figure 8.

At this stage we can already state with certainty that welfare of the supply side of the market has fallen down, as we know that net effect of the reform for unemployed workers is negative (Figures 5-6). The only interesting question here is just: "Who loses more?" Figure 9 provides the answer.

Figure 9: Relative welfare loss (supply side)

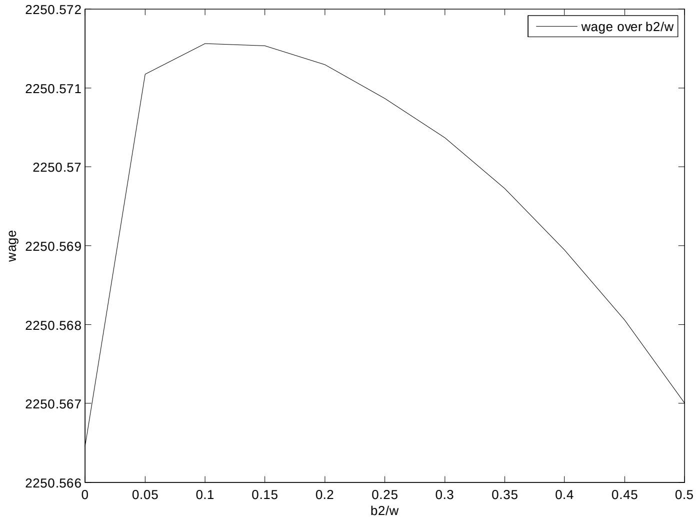


We know that the value of unemployment is the highest right at the entry, i.e. at $s = 0$. Figure 9 plots the ratio $V(b_1, 0)/V(w)$ against the replacement rate for unemployment assistance. The graph clearly shows that the reduction of assistance benefits leads to a larger welfare loss for unemployed workers if compared to employed ones.

Summing up, the reduction of unemployment assistance payments has led to a welfare loss of all parties on the supply side of the market, excluding thus any possibility of redistribution between employed and unemployed workers.

On the demand side of the market the situation is, in general, ambiguous. As we have discussed above, there are two counterforces that influence the equilibrium wage. Employment effect pushes the wage up and effort effect pulls it down. From equation (8) one can see that the dependence between the value of a firm and the wage is negative. So, firms lose if benefit reforms have positive net effect on wages, and vice versa. Figure 10 plots wages against the replacement rate. From this figure we can see that firms lose if benefit reduction is moderate, but gain if benefit cuts are substantial.

Figure 10: Equilibrium wage



This reaction pattern of the value of a firm also implies that for a substantial cut in benefits there may appear a possibility of redistribution between firms and workers, which may compensate individual welfare loss on the supply side.

Entitlement to UI benefits and individual welfare

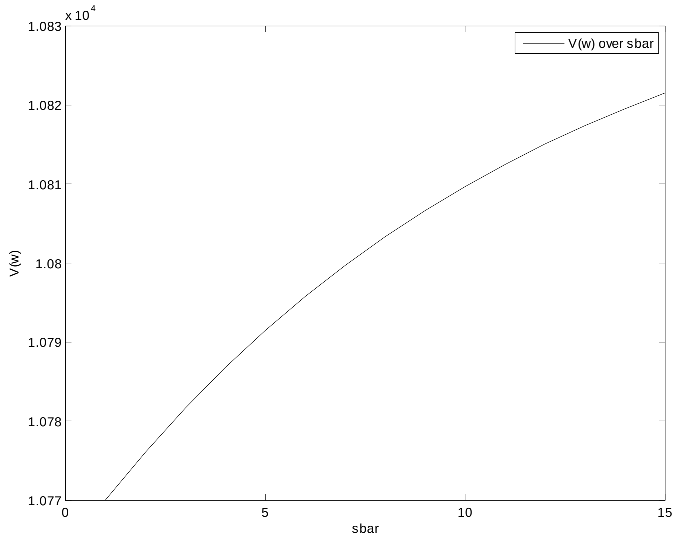
Unemployed workers unambiguously lose, because, as a result of the entitlement cut, some of them will need to start exerting higher effort levels earlier than before.

For employed workers the welfare consequence of a reduction in $\bar{s}$ can be tracked by the same argument we used when discussing Figure 8. Entitlement cut reduces the value of the outside option $V(b_1, 0)$ but is ambiguous for the wage. From Figure 11, however, we see that even if the net wage effect is positive, it still cannot offset the influence of the diminished value of the outside option.

So, again, supply side bears a welfare loss with no possibility of redistribution between employed and unemployed workers.

Influence of the entitlement reform on the value of the firm is ambiguous, because wage can both increase and decrease. Unfortunately at this stage of progress we do not have a graph similar to Figure 10, but with the entitlement length on the x-axis. However, the response pattern of the equilibrium wage is the same as in Figure 10 and for medium to big

Figure 11: Welfare of employed worker



reduction of $\bar{s}$ the value of a firm should go up. So, in general, entitlement reform should leave a scope for redistribution between supply and demand sides.

Summing up the results from the simulation of both types of reforms, we see that in all situations workers have incurred welfare losses. Firms may either gain or lose depending on the magnitude of assistance benefits and entitlement length reduction. So, in principle, the reform of Hartz IV type must provide a possibility of wealth redistribution. However, from Figure 10 this does not seem to be realistic in our particular example.

Our example considers only changes in the individual welfare. Though, pooling all our individual results together, it does not seem unlikely that the reform leads to a net social welfare loss.

6 Conclusion

In this paper we formulate an estimable equilibrium search model that extends the setting of Pissarides (1985, 2000) by introducing time-dependent stepwise unemployment benefits. Equilibrium solution of the model provides us with the endogenous distribution of unemployment duration characterized by a time-dependent hazard function. This result is new to the literature. Time-dependence of the exit rate from unemployment implies a Semi-Markov structure for the expected unemployment rate. Characterization of equilibrium unemployment rates undertaken in this paper is another methodological contribution to the literature.

On the empirical side, the model provides a unified framework for assessing performance of any element of the benefit system design, given that the system is stepwise. Empirical relevance of the model ranges from the analysis of a restructuring of a welfare state to the analysis of setting up one.

We estimate the model to look into welfare consequences of Hartz IV reform of unemployment benefit system in Germany. Our findings indicate that although the reform leads to a reduction of both aggregate and long-term unemployment, its welfare outcomes for representative employed and unemployed workers are negative. Welfare of employed and unemployed workers diminishes both with the reduction of unemployment assistance benefits and with the cut in the duration of entitlement to UI benefits. In general, the reform has an ambiguous effect on the value of a firm. However, in our particular case, the scope for wealth redistribution between firms and workers is, most likely, nonexistent.

Notes

[1]Specification of a similar kind can be followed back to Nickel (1979). However, unlike in Nickel (1979), we do not amend a stationray search model by assuming an ad hoc time dependence. Instead we introduce an additional exogenous time-dependent effect that complements our endogenous exit rate, the latter being already time-dependent. See p.15-p.16 for further discussion.

[2]As we see form (18), apart form wage w, value of employment $V(w)$ depends on $V(b_1, 0)$, which is the solution of (16) evaluated at the instant of entry into unemployment. Therefore to solve the household problem in practice, for every given wage w (and tightness θ) we need to take an initial guess for $V(b_1, 0)$, obtain initial value of $V(w)$ and given this value solve (16) to find a new $V(b_1, 0)$. This new value of unemployment at $s = 0$ will imply the new value of employment $V(w)$ and the cycle is repeated again and again until convergence is reached.

[3]The only possible source of error might remain in the value of the bargaining power β used to simulate the effect of reforms. However, one can easily repeat all comparative statics exercises with a variety of plausible values of β providing thereby wider confidence bounds for any predicted effect.

[4]As not uncommon in the empirical search literature, our model does not identify the rate of time preference. When estimating the model we set ρ equal to 0.003, which corresponds to the annual interest rate of 3.7%.

[5]Furthermore, the empirical model opens another avenue. Letting α vary free in presence of a non-benefit effect we can offer a simple test for the significance of the incentive mechanism of the benefit system as such. In case α is not significantly different from zero, the benefit mechanism does indeed influence the exit decision. This will provide a new, purely structural, view of the role of benefit system, clarifying the contradicting evidence (Hujer and Schneider, 1989, and Arulampalam and Stewart, 1995, vs. Carling et al.,

2001, and Røed and Zhang, 2003; see Introduction) and largely contributing to the reduced-form empirical literature. Estimation of this extension is currently underway.

[6]Please note that from this point onward the discussion will be based on the one-beofre-last estimation results. Consequently, all the conclusions we will draw here are inetrmediate.

References

ACEMOGLU, D. AND R., SHIMER, "Efficient Unemployment Insurance", *Journal of Political Economy* 107 (1999), 893-928.

ARULAMPALAM, W., AND M., STEWART, "The Determinants of Individual Unemployment Duration in an Era of High Unemployment", *Economic Journal* 105 (1995), 321-332.

ATKESON, A. AND R., LUCAS, "Efficiency and Equality in a Simple Model of Efficient Unemployment Insurance", *Journal of Economic Theory* 66 (1995), 64-88.

BLANCHARD, O., AND J., WOLFERS, "The Role of Shocks and Institutions in the Rise of European Unemployment: The Aggregate Evidence", *Economic Journal* 110 (2000), C1-C33.

CAHUC P., AND E., LEHMANN, "Should Unemployment Benefits Decrease with the Unemployment Spell", *Journal of Public Economics* 77 (2000), 135-153.

CARLING, K., HOLMLUND, B., AND A., VEJSIU, "Do Benefit Cuts Boost Job Findings", *Economic Journal* 111 (2001), 766-790.

FREDRIKSSON P., AND B., HOLMLUND, "Optimal Unemployment Insurance in Search Equilibrium", *Journal of Labor Economics* 19 (2001), 370-399.

FREDRIKSSON P., AND B., HOLMLUND, "Improving Incentives in Unemployment Insurance: A Review of Recent Research", *Journal of Economic Surveys* 20 (2006), 357-386.

HOPENHAYN, H. AND J-P., NICOLINI, "Optimal Unemployment Insurance", *Journal of Political Economy* 105 (1997), 412-438.

HUJER, R., AND H., SCHNEIDER, "The Analysis of Labour Market Mobility Using Panel Data", *European Economic Review* 33 (1989), 530-536.

KULKARNI, V., "Modeling and Analysis of Stochastic Systems", (Chapman & Hall, 1995).

LANCASTER, T., "The Econometric Analysis of Transition Data", (Cambridge University Press, 1990).

LJUNGQVIST L., AND D., SARGENT, "The European Unemployment Dilemma", *Journal of Political Economy* 106 (1998), p.514-550.

LJUNGQVIST L., AND D., SARGENT, "Two Questions about European Unemployment", *Econometrica* 76 (2008), p.1-29.

MORTENSEN, D., "Unemployment Insurance and Job Search Decisions", *Industrial and Labor Relations Review* 30 (1977), 505-517.

NICKEL, S., "Estimating the Probability of Leaving Unemployment", *Econometrica* 47 (1979), 1249-1266.

PISSARIDES, C., "Short-Run Equilibrium Dynamics of Unemployment, Vacancies and Real Wages", *American Economic Review* 75 (1985), 676-690.

PISSARIDES, C., "Equilibrium Unemployment Theory", (MIT Press, 2000).

POLYANIN, A., AND V. ZAITSEV, "Handbook of Exact Solutions for Ordinary Differential Equations", (Chapman & Hall/CRC, 2003).

RØED, K., AND T., ZHANG, "Does Unemployment Compensation Affect Unemployment Duration?", *Economic Journal* 113 (2003), 190-206.

ROSS, S., "Stochastic Processes", (Wiley, 1996).

SHAVELL, S. AND L., WEISS, "The Optimal Payment of Unemployment Insurance Benefits over Time", *Journal of Political Economy* 87 (1979), 1347-1362.

VAN DEN BERG, G., "Nonstationarity in Job Search Theory", *Review of Economic Studies* 57 (1990), 255-277.

Appendix

A: Data appendix

Below you can see the table with descriptive statistics

Table A1: "Descriptive Statistics"

Unemployment:	Mean	Std. Dev.	**Employment** [a]:	Mean	Std. Dev.
Duration (s)	11.21	14.09	Duration (l), cens.	61.25	28.66
UI benefits (b_1)	1357.04	508.12	Duration (l), all	42.80	31.98
UA benefits (b_2)	709.24	624.17			
Entitlement ($\bar{s}$)	15.84	7.49			
Last wage (w)	2250.57	901.79			
# obs., censored	17		# obs., censored	159	
# obs., total	316		# obs., total	325	

[a] Entrants to employment only

More information about the data is coming soon.

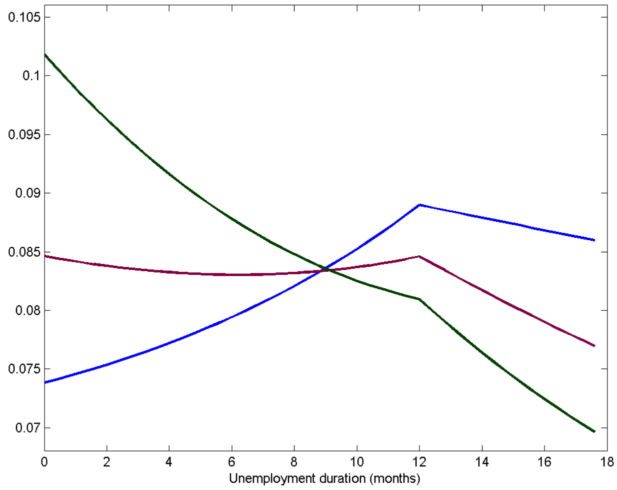
B: Structural hazard functions

The two figures below demonstrate flexibility of our structural hazard functions.

Exponential non-benefit effect, $\eta(s) = \eta_1 \exp\{-\delta_1 s\} + \eta_2$:

Even with a simple exponential specification the model generates non-monotone hazard functions. Blue line in Figure 12 plots the hazard function with a relatively strong benefit effect. Search intensity, and so the hazard, increases up to the date of the expiration of entitlement. Beyond this date benefit effect disappears, an the exit rate does down. Once non-benefit effect grows stronger and stronger, the exit rate before $\bar{s}$ becomes flatter and flatter. Purple line shows the case, in which benefit and non-benefit effect are equally strong, offsetting each other and making the risk of exit before the expiration of entitlement nearly independent of unemployment duration. Again, beyond $\bar{s}$ hazard rate goes down. Finally olive line shows a relatively strong complementary effect, which leads to a monotone decreasing hazard function throughout.

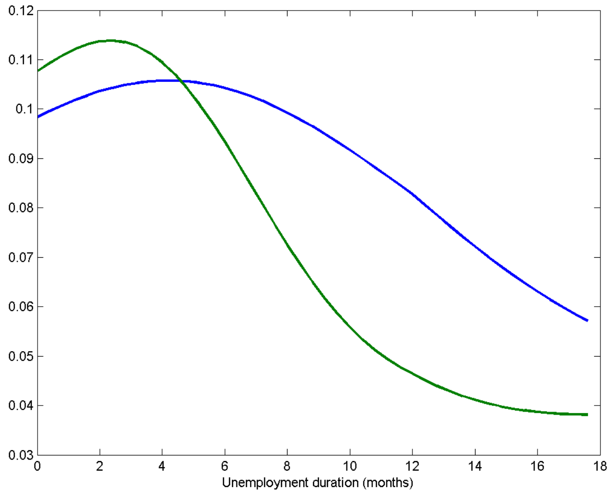
Figure 12: Structural hazards - I



Weibull non-benefit effect, $\eta(s) = \eta_1 \exp\left\{-\delta_1 s^{\delta_2}\right\} + \eta_2$:

Weibull specification further generalizes the hazard function, making it both convex and concave. Figure 13 illustrates the case.

Figure 13: Structural hazards - II



C: Derivations and proofs

Selected derivations and proofs here ... *(everything you see is already derived and proven)*